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Convertible Bonds and Bank Risk-Taking

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Convertible Bonds and Bank Risk-Taking

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Abstract

We study how contingent capital affects banks' risk choices. When triggered in highly levered states, going-concern conversion reduces risk-taking incentives, unlike conversion at default by traditional bail-inable debt. Interestingly, contingent capital (CoCo) may be less risky than bail-inable debt as its lower priority is compensated by a lower induced risk. The main beneficial effect on risk incentives comes from reduced leverage upon conversion, while any equity dilution has the opposite effect. This is in contrast to traditional convertible debt, since CoCo bondholders have a short option position. As a result, principal write-down CoCo debt is most desirable for risk preventive purposes, although the effect may be tempered by a higher yield. The risk reduction effect of CoCo debt depends critically on the informativeness of the trigger. As it should ensure deleveraging in all states with high risk incentives, it is always inferior to pure equity.

Keywords: Banks; Contingent Capital; Risk-shifting; Financial Leverage;

JEL Classifications: G13, G21, G28.

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1 Introduction

During the recent credit boom, bank capital had fallen at historical lows. In the ensuing crisis, banks could not absorb losses, leading to credit market disruption and spillovers to the real economy. The regulatory response has focused on higher core bank equity to absorb losses and protect depositors. While Basel III core capital ratios require common equity, additional buffers (required for instance for SIFIs, systematically important financial institutions) may include bail-inable and convertible debt. Issuance of bank convertible debt has risen rapidly to over 200 billion dollar in the last five years.

Originally proposed by Flannery (2002) and discussed extensively by Kashyap et al. (2008), contingent convertible capital (henceforth CoCo debt) is a debt claim that converts into equity or is partially written off when bank capital falls below some trigger level. Contingent deleveraging creates a buffer that reduces financial distress and deposit insurance losses (Pennacchi, 2011; Pennacchi et al., 2011; Albul et al., 2013; McDonald, 2013). A key question is whether CoCo debt shares with common equity the effect of increasing "skin in the game", attenuating incentives for risk-taking. The view is endorsed by many authors (Madan and Schoutens, 2010; Glasserman and Nouri, 2012; Koziol and Lawrenz, 2012; Albul et al., 2013; Hilscher and Raviv, 2014; Berg and Kaserer, 2014; Chan and van Wijnbergen, 2016), though this conclusion follows only as a comparative statics exercise on the sensitivity of equity value to exogenous changes in asset risk.[1]

The paper focuses on how contractual features of CoCo debt affect a bank's incentives to take inefficient risk that involves a choice for speculative assets purely for risk shifting reasons, as they would not be chosen by an unlevered investors. To our knowledge, this is the first treatment of banks' endogenous asset risk choice in response to CoCo debt.[2]

While equity would naturally offer the lowest risk incentives, our starting point presumes that CoCos are issued because an increase in bank equity is not an option.[3] We show that going-concern CoCo capital reduces risk-taking as it forces deleveraging in highly levered states, when risk-shifting incentives are strongest.

Our structural analysis reveals some surprising effects of CoCo debt's contractual features. [4]

[1]Partial exceptions are Chen et al. (2013), where equityholders may choose for strategic default when insolvent, and Chan and van Wijnbergen (2016) who study a bank's trade-off between probability of conversion and bankruptcy cost.

[2]We offer an overview of the main assumptions and results in the literature in the Appendix.

[3]Distinct benefits of CoCos versus equity are its lower after-tax cost and better incentives for effort under a managerial agency conflict (Kashyap et al., 2008).

[4]Consistently with a scarcity of bank equity, the trigger on CoCo debt is set so as to avoid conversion in states when it does not contribute to reducing risk-taking.

A key result is that CoCo debt can be shown to be better at reducing ex ante risk than conventional bail-inable debt. Going-concern conversion in high leverage states reduces risk-shifting, while debt bail-in only reallocates losses at default among debtors. As it does not affect shareholder payoffs, it does not alter their ex ante risk incentives.

The main contribution to endogenous risk reduction turns out to be the direct effect of leverage reduction upon conversion. In contrast, equity dilution caused upon a going concern conversion does not reduce inefficient risk-taking but actually enhances it. This is in contrast with the result on risk incentives under conventional convertible debt Green (1984). The insight is that traditional convertible bondholders have a long option position and convert only in a favorable outcome, diluting high returns from riskier choices. Instead, CoCo debt automatically turns into equity when asset values are revealed to be low. [5] Equity dilution upon conversion has a counterproductive effect because it reduces the share of value accruing to shareholders of a safer asset choice. Thus a main conclusion is that CoCo debt reduces risk-taking via its debt reduction effect, not through its equity dilution. A separate debt reduction effect arises because a fixed conversion ratio implies a value transfer from CoCo holders to equityholders, which we term CoCo dilution (Berg and Kaserer, 2014). [6] In practice, such a value transfer to equity occurs in all existing CoCo debt issues, as it is impossible to fully adjust the conversion by a contingent conversion ratio (in some states even an infinite dilution would be insufficient). The CoCo dilution has the effect of reducing equity dilution and thus also banks' risk-shifting incentives. In the extreme case of principal write down CoCo debt, the equity dilution effect is minimized to zero.

A final consideration is that as CoCo debt has on average a lower priority than other forms of debt it may be expected to command a higher yield, which increases effective leverage and thus risk incentives. However, as CoCo bonds induce less risky choices they may be safer in equilibrium, in which case they would carry a lower yield.

We next compare the two main forms of CoCo debt conversion, namely conversion to equity versus principal write-down. Our results suggest that a debt write-off upon conversion should be preferred, since debt dilution has a favorable effect and equity dilution has a negative effect on risk incentives. In general however write-down CoCo debt may have a higher yield if it suffers larger loss at conversion. A higher yield worsens risk choice, an effect that must be traded off against the balance of the equity and CoCo dilution effects. [7] Numerical calibrations

[5] In the model, conversion has a preventive function as it occurs in a timely fashion when risk choices can still be influenced. A delayed conversion just before default would clearly not influence incentives.

[6] Berg and Kaserer (2014) study how CoCo debt dilution affects equity valuation near the trigger to assess the effect of any value transfer to and from CoCo debt.

[7] It is easy to see that if the yield were the same for both types, a principal write-down version always induces a

suggest that although converted to equity CoCos are more expensive than principal write-down CoCos, the risk reduction effect of the latter is usually stronger. The intuition is that a contingent conversion reduces endogenous risk sufficiently to compensate for their larger risk bearing.

The effectiveness of CoCo conversion on risk incentives critically depends on the precision of the trigger, and specifically how frequently it delivers deleveraging in states when risk-shifting incentives have deteriorated. Equity is equivalent to conversion in all states and therefore always ensures safer choice. A final conclusion is that CoCo debt will fail to control risk in the most levered banks, for which there is a clear role for direct regulatory intervention.

1.1 Model discussion and related literature

Our setup offers a limited welfare analysis. We take initial capital requirements and deposit insurance as given. Following the literature, insured deposits are assumed to be the cheapest form of funding as deposit insurance is not properly priced. [8]. We also do not seek to explain the amount of CoCo issued (although our calibration model may be used to optimize the tradeoff between a higher equity content and a higher yield). Rather than introducing an arbitrary rationale for the choice of CoCo debt versus equity, we implicitly presume that there is a scarcity of equity investors or an unsurmountable reluctance by banks to issue more equity. Accordingly, in the model the trigger level is set to economize on equity content by avoiding any CoCo conversion for banks for which it would not improve risk incentives. This choice could be rationalized in a setting with an increasing cost of risk absorption capacity and a regulatory goal to minimize the frequency of bank default, a framework we plan to adopt in future work.

In our basic model CoCos act as a junior debt upon default at maturity rather to convert in equity. This feature turns out to be desirable as it reduces risk incentives by avoiding a transfer to old shareholders in near-default states. In an extension we consider a more conventional CoCo conversion into equity upon default. In this case a riskier asset choice at the interim date increases the chance of conversion (as it is the case with other models where conversion may occur in continuous time). Gone-concern conversion is shown to induce riskier choice for some banks that were previously playing safe (since it increases the chance of a value transfer from CoCo debtholders), while some highly levered banks may now be safer precisely to ensure a value transfer. We show that on average the proper trigger value should be set higher.

safer choice.

[8]Deposit insurance charges would ceteris paribus increase effective leverage. While the low cost of deposits has a positive effect on risk incentives, we do not advocate subsidizing bank liability risk. Sundaresan and Wang (2014) show how even fairly priced insurance will induce banks to take excess risk.

In line with the literature, the trigger is based on an observable value of bank assets. All outstanding CoCo issues are designed to be triggered by accounting value triggers, which implies that they may convert only at discrete intervals, as in our model. Regulators have not encouraged continuous time triggers based on market prices, as they may be manipulable and cause multiple pricing equilibria (Sundaresan and Wang, 2015), a finding challenged by Pennacchi and Tchistyi (2015). In practice both market and accounting measures are imprecise and manipulable. Other approaches to conversion are offered in Duffie (2010) and McDonald (2013), where the case of conversion in a systemic crisis is examined. Glasserman and Nouri (2012) study the interesting theoretical case of a continuous conversion feature. Chan and van Wijnbergen (2014) highlight a possible systemic effect of CoCo conversion, as it may trigger runs when asset risk is correlated across banks.

In our set up (as in all outstanding CoCo debt), conversion never results in a positive value transfer to CoCo holders. Hilscher and Raviv (2014) argue that a very high equity dilution rate at conversion may discourage risk-taking. In practice, converting debt into shares at below book value faces serious legal issues, and is impossible in states when net equity is negative.

Our approach has implications for pricing of contingent capital. CoCos are usually priced as a package of conventional bonds and a short position in a put option. This leads to underpricing as it neglects their risk-reducing effect, which reduces the value of their short option position.

There is still limited empirical work on CoCo pricing and their effect on bank risk. An issuance of CoCo debt should reduce the overall chance of bank default. Interestingly, Avdjiev et al. (2015) show that the market impact of a bank issuing CoCo debt is a lower CDS spread, but has no effect on share returns. This combination of results suggests that issuing CoCo not only absorbs risk but is also associated with lower expected asset risk.[9] The introduction of CoCo debt may also be beneficial by inducing voluntary equity recapitalization to avoid costly dilution of profits or loss of control, as shown in Pennacchi et al. (2011) and Calomiris and Herring (2013).

Section 2 presents the basic setup, and Section 3 shows how CoCo design affects the banker's risk choice. Section 4 prices different contractual forms and compares the yield among bail-inable debt, principal write-down CoCos and converted to equity CoCos and compares their effect on risk choice under endogenous yield. Section 5 considers CoCos that may convert at maturity and compares their risk reduction effect with those that act at maturity as a junior debt. Section 6 compares the bank risk choice under CoCo versus equity funding. Section 7 concludes. The Appendix contains all the proofs as well as an overview of the main assumptions and results

[9] Avdjiev et al. (2015) suggest that current CoCo prices do not properly reflect losses at conversion, represented in our setting by the CoCo dilution effect.

in the literature.

2 Model Setup

All agents are risk-neutral. There are three dates: $t = 0$, $t = 1$ (interim date) and $t = 2$. The bank has initial assets of value 1 at $t = 0$, financed with $1 - D$ of equity that satisfies an exogenous capital requirements. Bank debt D may include CoCo bonds with face value C and insured deposits $D - C$.[10] The interim CoCo coupon rate, the deposit rate and the deposit insurance premia are normalized to zero. CoCo debtholders are competitive, so they break-even in expectation. There is one active agent in the model: the banker/bank owner, who chooses the interim level of credit risk.[11] The banker's payoff is the value of the original bank equity at $t = 2$. Borrowers are price-takers.

At $t = 1$, the asset value v is uniformly distributed over $[1 - \delta, 1 + \delta]$, so interim leverage is $\frac{D}{v}$. This is initially observed only by the banker, who then chooses whether to exert risk control effort ($e = 1$) or not ($e = 0$). Effort is costless.

Depending on the banker's choice, asset values at $t = 2$ may be safe or risky. If the banker exercises risk control, assets produce a safe payoff with gross return 1, and the final asset value is v. Alternatively, a risky credit strategy has payoff $V_2 = v - z + \sigma Z$, where Z has zero mean and variance 1. Thus, the mean of the final asset value is $\mathbf{E}(V_2) = v - z$, and the standard deviation is σ, distribution function is $F(V_2)$ and density function is $f(V_2)$. Critically, we assume $z > 0$, so the riskier strategy is a dominated asset that yields less on average than the safer asset.[12] However, when debt exceeds asset values, a risky choice would enhance equity value.

After the risk choice is made at $t = 1$, the value v is revealed with probability φ to all investors. (Note that φ measures the precision of public information.) We assume that no bank equity may be raised at this time. This signal is verifiable and may be used as a trigger for conversion. CoCos are automatically converted into equity when the interim asset value v is revealed to have fallen below a pre-specified trigger level v_T at $t = 1$. The trigger value is initially set lower than the initial book value 1, else there would be immediate conversion at $t = 0$. As we study going-concern conversion, we assume that at $t = 1$ the value of equity is positive upon conversion of CoCo debt, so $v \geq D - C$ for any v. After conversion, the

[10] We assume that it is mandatory for the bank to issue CoCos.

[11] We assume the bank manager is the sole equityholder, to focus on risk-taking rather than on other agency conflicts between managers and equityholders.

[12] As a result, the distribution of asset return in the safe outcome has second-order dominance relative to risky outcome, though not first-order dominance.

amount of shares is $1+d$, where d is the amount of shares CoCo holders get upon conversion, equal to the ratio of face value of CoCos over the trigger asset value minus debt: $d = \frac{C}{v_T - D}$. This conversion ratio ensures no value transfer when asset value equals exactly v_T, but leads to a transfer from CoCo holders as soon as v is strictly below v_T which we refer to as CoCo dilution.[13]

At $t = 2$, CoCos repay $C(1+y)$, where y is initially exogenous (we endogenize the yield in Section 4). To highlight the pure effect of going concern conversion, we assume that at $t = 2$ CoCos act as traditional junior debt, senior to equity.[14]

The current payoff structure is presented in the Figure 1. The trigger value v_T is set by a

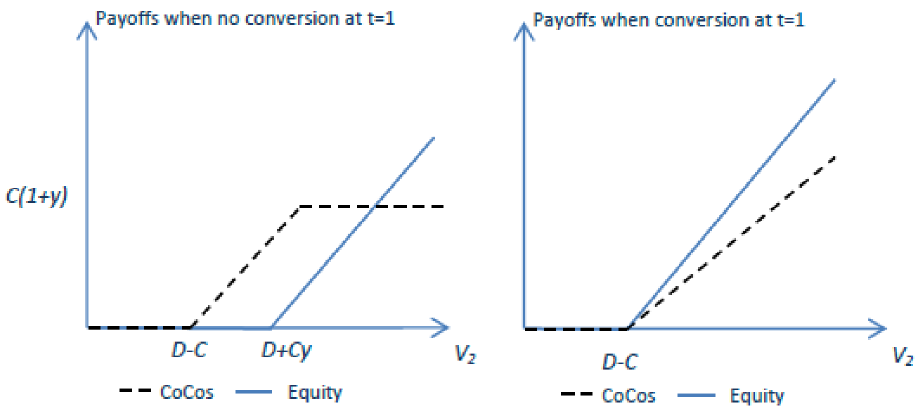

Figure 1: Payoffs in case of no conversion vs early conversion ($d < 1$)

regulator whose objective is to minimize ex ante risk (i.e. the probability that the banker chooses the inefficient risky strategy), while avoiding unnecessary conversions.

The sequence of events is presented in Figure 2.

3 Bank Debt Design and Risk-Shifting Incentives

The banker makes her risk decision to maximize her expected payoff upon observing v. Her risk shifting incentives can be characterized in terms of the range of values for which risk control is chosen. Under our assumption of going concern conversion $v \geq D - C$, when risk is controlled

[13]Conversion at par via a contingent conversion ratio is possible only over some range, but in low states even a conversion in an infinite amount of shares would not be sufficient. Full equity dilution ahead of default would in any case be illegal.

[14]In Section 5 we consider CoCos that can convert into equity at maturity.

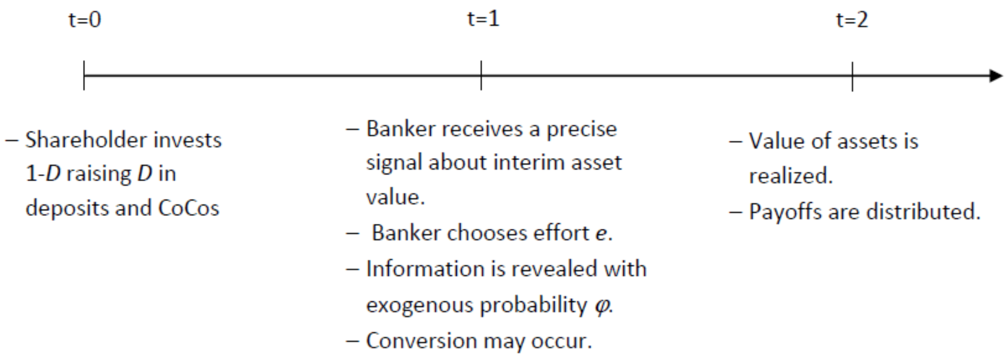

Figure 2: The sequence of events

depositors are always repaid from bank assets, even in default. In contrast, if the banker chooses the risky asset, losses may fully wipe out CoCo debt and force some losses on the deposit insurance fund.

3.1 Baseline Case: Deposits

Consider the baseline case when instead of CoCo debt of amount C bank holds deposits, and thus all debt D is represented only by deposits. We follow the literature in assuming that deposit insurance not properly priced, so that deposit funding is the cheapest form of debt.

The expected banker's payoff from a risky asset choice is:

$$(1 - F(D|v)) \cdot \mathbf{E}(V_2 - D|V_2 > D, v) = \int_D^\infty (V_2 - D) f(V_2|v) dV_2 \tag{1}$$

which is the sum of its unconditional mean $\mathbf{E}(V_2 - D) = v - z - D$ (which may be negative) and a measure of the right tail return in solvent states:

$$\begin{aligned}\int_D^\infty (V_2 - D) f(V_2|v) dV_2 &= v - z - D - \int_{-\infty}^D (V_2 - D) f(V_2|v) dV_2 \\ &= v - z - D + \Delta(v, D, \sigma) \end{aligned} \tag{2}$$

where $\Delta(v, D, \sigma)$ is the value of the put option (commonly called Merton's put) enjoyed by bank equityholders under limited liability. It measures the temptation of the banker to shift risk, defined as the difference in her payoff between a risky and safe strategy:

$$(1 - F(D|v)) \cdot \mathbf{E}(V_2 - D|V_2 > D, v) - (v - D) = -z + \Delta(v, D, \sigma) \tag{3}$$

We henceforth refer to $\Delta(v)$ as the measure of risk-shifting incentives.

Lemma 1. *If the risky asset payoff is normally or uniformly distributed, risk-shifting incentives $\Delta(v)$ are monotonically decreasing and convex in asset value v: $-1 \leq \Delta'(v) \leq 0, \Delta''(v) \geq 0$. Moreover, risk-shifting incentives increase with a higher volatility of risky asset σ.*

Thus, for a normal distribution, risk-shifting incentives are given by:

$$\Delta(v) = (v - D - z) \cdot \left[\Phi\left(\frac{v - D - z}{\sigma}\right) - 1\right] + \sigma \cdot \phi\left(\frac{v - D - z}{\sigma}\right) \tag{4}$$

We assume that $f(V_2)$ is such that the risk-shifting incentive function is convex in asset value.

Assumption 1. *Risk-shifting incentives $\Delta(v)$ are decreasing and convex function of asset value v: $-1 \leq \Delta'(v) \leq 0, \Delta''(v) \geq 0$. Also $\Delta(v)$ are increasing with σ: $\Delta'(\sigma) \geq 0$.*

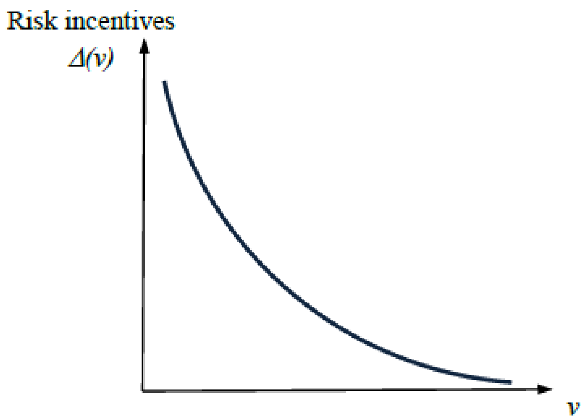

Figure 3: Risk-shifting incentives under Gaussian risk distribution

Now consider the risk choice of the banker when bank's debt is represented only by deposits. The banker's program is:

$$\max_e e \cdot \underbrace{\max(v - D, 0)}_{\text{equity value if } e=1} + (1 - e) \cdot \underbrace{(v - z - D + \Delta(v))}_{\text{equity value if } e=0}$$
$$\text{s.t. } e \in \{0, 1\} \tag{5}$$

At $\Delta(v) = z$ the net present value of the banker's choice of a risky lending strategy is zero. Under the Assumption 1, the banker's choice is:

$$e = \begin{cases} 1 & \text{if } v \geq \max\left[\Delta^{-1}(z), D\right] \equiv v_D^* \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where v_D^* is the cut-off interim asset value.

Lemma 2. *Under deposit funding, the banker controls risk if interim leverage is sufficiently low, namely if $v \geq v_D^* \equiv \max\left[\Delta^{-1}(z), D\right]$. If $\Delta^{-1}(z) > D$, threshold v_D^* rises and the ex ante probability of risk control (defined as $\frac{1+\delta-v_D^*}{2\delta}$) decreases with the volatility of risky asset σ.*

We now use the threshold v_D^* as a measure of risk prevention against which different forms of bank debt may be compared.

3.2 Bail-inable debt

In our setting CoCo debt resembles bail-inable debt at maturity date $t = 2$ unless it converts at $t = 1$. To establish the effect of the convertibility feature, we first derive the benchmark case of pure bail-inable debt.

Consider that instead of CoCo debt of amount C bank holds bail-inable debt, promising a yield Cy at $t = 2$. As it is never converted, it is equivalent to CoCo debt when conversion is never triggered (such as when there is never an informative signal $\varphi = 0$).

Then the banker's program is:

$$\begin{aligned} &\max_e e \cdot \max\left(v - D - Cy, 0\right) + (1 - e) \cdot (v - z - D - Cy + \Delta(v - Cy)) \\ &\text{s.t. } e \in \{0, 1\} \end{aligned} \tag{7}$$

where $\Delta(v - Cy) = -\int_{-\infty}^{D+Cy} (V_2 - D - Cy) f(V_2|v) dV_2$ is its risk-shifting incentive under bail-inable debt.

The optimal risk choice by the banker is to control risk if $v \geq v_D^* + Cy \equiv v_B^*$.

Proposition 1. *(Yield effect) Under bail-inable debt funding, the banker controls risk less often than under deposits, since $v \geq v_B^* \equiv v_D^* + Cy$. The frequency of risk control $\frac{1+\delta-v_B^*}{2\delta}$ declines with yield y and the amount of bail-inable debt C.*

This result simply reflects the lower effective leverage associated to deposits because of their lower yield, which is solely due to deposit insurance. The resulting higher leverage leads to worse risk control. This *yield effect* is shown in Figure 4 as the difference between v_B^* and v_D^*.

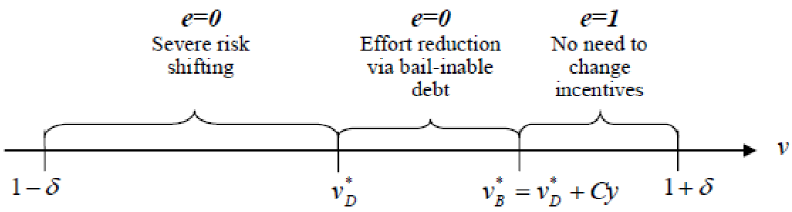

Figure 4: Risk choice under bail-inable debt

Note that in both cases of deposit or bail-inable debt, an interim revelation of v has no effect on risk choice, as this disclosure does not change leverage.[15]

3.3 CoCo Debt

This section considers the case of an amount C of CoCo debt, triggered by a public signal with precision φ. An efficient design of CoCo debt requires that a trigger improves banker's risk control decision. Intuitively, conversion should be triggered when bank's interim leverage is high enough. We set the trigger such that no conversion occurs for well-capitalized banks which have low risk incentives. This choice trades-off the frequency of risk control against an increasing marginal cost of risk absorbing capital.

3.3.1 Trigger value

We first consider the banker's risk control decision under a given trigger v_T, and use the result to define the proper trigger level.

From Proposition 1, setting a trigger higher than v_B^* does not change risk control choice for low levered banks (defined as those with $v \geq v_B^*$). Moreover, the range of trigger values is the interval satisfying $v_T < 1$ to ensure no conversion at $t = 0$. Upon conversion, CoCo holders get the amount of shares $d = \frac{C}{v_T - D}$, which we refer to as conversion ratio. CoCo conversion dilutes shareholders' payoff to $\frac{1}{d+1}$ share of equity. Further we show that equity dilution induces riskier choices as it reduces the value the shareholders get under safe strategy. Such a conversion ratio

[15] As depositors are insured, they would not run at $t = 1$ even if v is low.

also ensures CoCo debt dilution, i.e upon conversion holders of CoCo debt get less than its face value. CoCo dilution enhances safer choice since it has the opposite effect than equity dilution by increasing shareholders' stake.

Consider the banker's program:

$$\max_{e} e \cdot [\underbrace{\max(v - D - Cy, 0) \cdot (I(v \geq v_T) + (1-\varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=1} +$$
$$\underbrace{\frac{v - D + C}{d+1} \cdot \varphi \cdot I(v < v_T)]}_{\text{equity value if conversion and } e=1} +$$
$$(1-e) \cdot [\underbrace{(v - z - D - Cy + \Delta(v - Cy)) \cdot (I(v \geq v_T) + (1-\varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=0} +$$
$$\underbrace{\frac{v - z - D + C + \Delta(v + C)}{d+1} \cdot \varphi \cdot I(v < v_T)]}_{\text{equity value if conversion and } e=0}$$
$$\text{s.t. } e \in \{0, 1\} \tag{8}$$

where $I(\cdot)$ is an indicator function, $\Delta(v + C) = -\int_{-\infty}^{D-C} (V_2 - D + C) f(V_2|v) dV_2$ is risk-shifting incentive under CoCos conversion.

This identifies two critical interim asset values. First, v_B^* is the value threshold at which the banker chooses for risk control even if no conversion may take place, equivalent to the threshold for bail-inable debt. Second, v_C^* is the risk control threshold for CoCo debt. This value depends on whether the critical asset value v_C^* is above the threshold $D + Cy$, in which case bank equity is positive even without conversion:

$$\varphi \cdot \frac{z - \Delta[D + C(1+y)]}{d+1} + (1-\varphi)(z - \Delta(D)) < 0 \tag{9}$$

where v_C^* is defined implicitly in (10):

$$\begin{cases} F = \varphi \cdot \frac{z - \Delta[v+C]}{d+1} + (1-\varphi)[z - \Delta(v - Cy)] = 0 & \text{if (9) holds} \\ G = \varphi \cdot \frac{z - \Delta[v+C]}{d+1} + (1-\varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \tag{10}$$

Proposition 2. *The introduction of CoCos improves risk control choice for banks relative to traditional bail-inable debt in the range of asset values $v_C^* \leq v \leq v_T$. Banks with extremely high leverage ($v < v_C^*$) still choose to take risk, while high value banks ($v > v_B^*$) choose to control risk.*

Banks with $v < v_C^*$ have such high leverage that they choose to gamble and not control risk.[16] Figure 5 shows how the risk control choice of the banker depends on the trigger, and in particular how it may not be monotonic in the interim asset value.

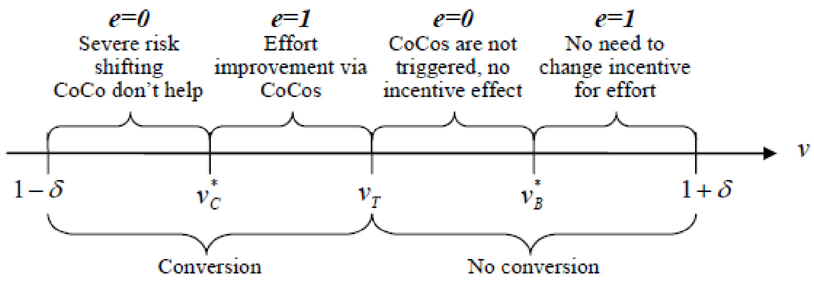

Figure 5: Risk choice under CoCos

The difference $\frac{v_T - v_C^*}{2\delta}$ measures the expected improvement in risk control induced by CoCos. It is easy to see that v_C^* is in the range $[v_D^* - C, v_B^*]$ and decreases with the probability of information revelation φ (see Figure 6).

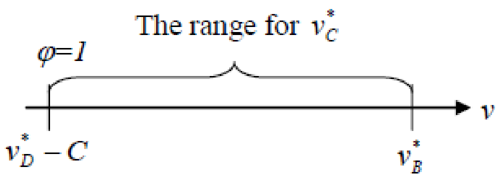

Figure 6: Cut-off value v_C^*

Lemma 3. *CoCo debt improves bank's risk control more as the trigger precision φ goes up.*

The intuition is that a more informative trigger forces more frequently a recapitalization in states with high risk-shifting incentives.

Having characterized the banker's risk control choice under a given trigger, we can identify the trigger level, which ensures the monotonicity of risk control choice. Setting the trigger value at $v_T = v_B^*$ ensures monotonicity of risk control choice and thus produces expected risk reduction $\frac{(1+\delta - v_C^*)}{2\delta}$ for a given amount of CoCo debt C (and thus a given conversion ratio d).

[16] Were CoCo debt large enough ($v_C^* < 1 - \delta$), this range does not arise, and all banks with $v < v_T$ have incentives to contain asset risk.

Figure 5 shows that unless the trigger v_T is chosen properly, the risk control choice is not necessarily monotonic in v. If the trigger is too high (above v_B^*), CoCos will not affect risk choice of low levered banks with $v > v_B^*$. If it is too low (below v_B^*), there will be no conversion for an intermediate range of highly levered banks. This is clearly inefficient, as it is easier to induce risk control for higher v. As a result, setting the trigger to $v_T = v_B^*$ guarantees the monotonicity of risk control with respect to asset value, as shown in Figure 7.

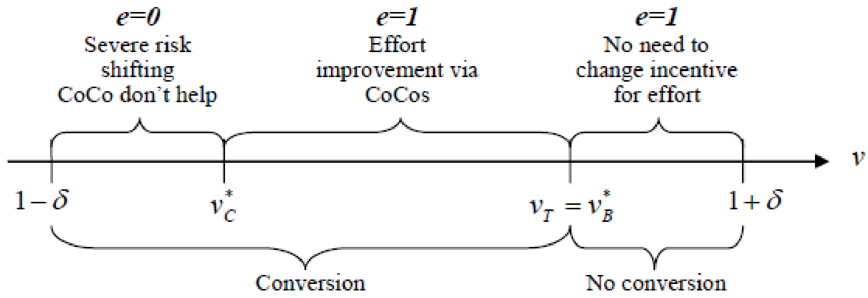

Figure 7: Risk choice under CoCos with restricted trigger asset value $v_T = v_B^*$

Lemma 4. *Under the trigger $v_T = v_B^*$, CoCos induce safer choice than bail-inable debt.*

Under the monotonic trigger, the risk control improvement under CoCos against bail-inable debt ranges from 0 (if $v_C^* = v_B^*$) to $\frac{v_B^* - (v_D^* - C)}{2\delta}$ (if $v_C^* = v_D^* - C$). Therefore, CoCos ensure better risk control than bail-inable debt. However, CoCo debt does not generally improve risk control relative to insured deposit funding whose yield is subsidized.

Lemma 5. *A higher amount of CoCo debt C or a higher yield y require that the trigger value be raised to maintain the monotonicity of risk control choice.*

As in the case of bail-inable debt, a higher promised yield at maturity Cy increases leverage and thus willingness to take risk. A higher trigger value $v_T = v_B^*$ here is needed to offset this effect subsidized by increasing the CoCo's equity content. However, it also leads to more frequent conversion and more risk bearing for CoCo debtholders. The next section endogenizes the yield for a general characterization.

The trigger value should be set higher when the risky asset choice is more volatile. A higher asset volatility increases the return to risk-shifting when $\Delta(D) > z$.

Having mapped the banker's risk control decision under a properly set trigger, we turn to decompose the effect of different CoCo debt features on bank's risk choice.

3.4 Forms of CoCo debt

3.4.1 Write-down CoCos

First we consider the extreme case when conversion leads to a full write-down of principal. When triggered, this type of CoCo debt "disappears". This is equivalent to the case of a zero conversion ratio $d = 0$.

When write-down CoCos convert, the bank has suddenly less leverage while shareholders keep all shares. This is an extreme form of CoCo dilution. The banker's program is similar to (8) under $d = 0$. As before, there is a critical value v^*_{WD} for the interim asset value above which the introduction of write-down CoCos improves risk control (see Figure 8).

Proposition 3 (Debt reduction effect)**.** *Under write-down CoCos with conversion ratio $d = 0$, the banker controls risk if $v \geq v^*_{WD}$. This improves risk control relative to converted to equity CoCo debt, since the cut-off value v^*_{WD} is lower than v^*_C.*

Intuitively, when conversion lowers bank leverage, risk incentives are reduced. The risk reduction relative to bail-inable debt represents a pure *debt reduction effect* represented as $\frac{v^*_B - v^*_{WD}}{2\delta}$ in Figure 8.

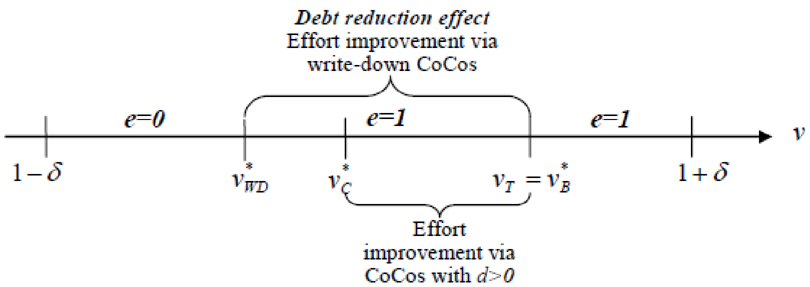

Figure 8: Risk choice under write-down CoCos. Debt reduction effect.

3.4.2 Equity conversion CoCos

Next we establish the benchmark of CoCos converted at par, and then discuss more realistic case of CoCos with the fixed conversion ratio.

Conversion at par

We briefly consider the possibility that the CoCo conversion ratio could be set such that no value transfer would occur upon conversion, i.e $d = \frac{C}{v-D}$. In other words, the CoCo bond payoff upon conversion is exactly equal to its face value C. This feature is impossible in practice for very low v, but it is considered here as it sets a useful benchmark.

The banker's program and its solution are equivalent to (8) as well as (9) and (10) respectively under $d = \frac{C}{v-D}$. There is a critical value v_P^*, the interim asset value above which the introduction of at par CoCos improves risk control.

Proposition 4 (Equity dilution effect)**.** *Under CoCos converted at par with conversion ratio $d = \frac{C}{v-D}$, the banker controls risk if $v \geq v_P^*$. Risk control is lower than for write-down CoCos, since the cut-off value v_P^* is higher than v_{WD}^*.*

Higher conversion ratio d implies higher equity dilution (the banker gets $\frac{v-D}{v-D+C} < 1$ of the total equity upon conversion). Since upon conversion net equity value is still positive $v > D-C$, a greater equity dilution reduces the net payoff to safe asset choice thus discouraging risk control. In other words, such a conversion increases the risk control threshold and thus reduces bank's incentives to control risk. We call the banker's risk control reduction from at par CoCos relative to write-down Cocos an *equity dilution effect* presented as $\frac{v_P^*-v_{WD}^*}{2\delta}$ in Figure 9. Note that by construction there will be no counteracting effect from CoCo dilution at conversion.

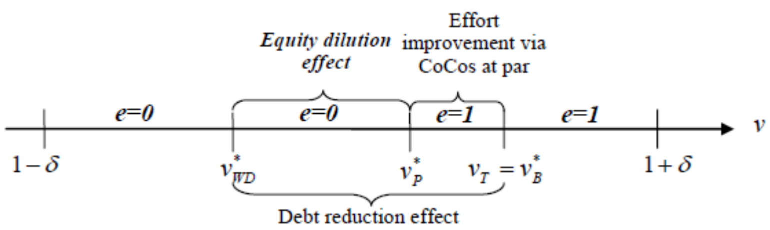

Figure 9: Risk choice under at par CoCos. Equity dilution effect.

Conversion with CoCo debt dilution

Conversion at par is not always possible, and it is never attempted in practice. We consider now the realistic case of going concern CoCos with fixed conversion ratio $d = \frac{C}{v_T-D}$, a design feature common to all outstanding CoCo bonds.

The banker's program and its solution are (8) as well as (9) and (10) respectively under $d = \frac{C}{v_T-D}$. The critical value is v_C^* above which the introduction of converted to equity CoCo debt improves risk control.

Proposition 5 (CoCo dilution effect)**.** *Under CoCos with fixed conversion ratio* $d = \frac{C}{v_T - D}$*, the banker controls risk if* $v \geq v_C^*$*. Risk control is higher than for CoCos converted at par, since the cut-off value* v_C^* *is lower than* v_P^**.*

This result identifies the CoCo dilution effect, driven by the value transfer from CoCo debt to equity upon conversion. Since the fixed conversion ratio is lower than the par conversion ratio $\frac{C}{v_T - D} < \frac{C}{v - D}$, upon conversion holders of CoCo debt get less than the promised face value. As a result, the equity dilution effect is reduced, and the safe asset payoff increases relative to the risky one. Thus, we can conclude that this produces better risk control. We can call the banker's risk control increase from CoCos with a fixed conversion ratio relative to at par CoCos a pure *CoCo dilution effect* presented as $\frac{v_C^* - v_P^*}{2\delta}$ in Figure 10.

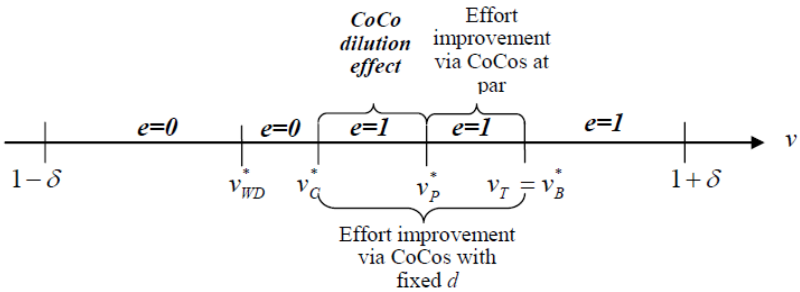

Figure 10: Risk choice under CoCos with fixed conversion ratio. CoCo dilution effect.

Note that fixed conversion ratio depends on the trigger value.

Lemma 6. *The value transfer from the holders of CoCo debt to equity holders (referred to as CoCo dilution effect) increases with the trigger value* v_T*, so CoCo debt with a higher trigger produces (ceteris paribus) greater risk reduction.*

Note that because of the CoCo dilution effect, a high trigger reduces risk-shifting incentives beyond the direct effect of their higher equity content. The key reason is that going-concern conversion ensures that equity value is positive under a safe strategy. Since risk control dominates the risky strategy, this improves the banker's risk choice.

Table 1 summarizes the effects of CoCos on the banker's risk choice under different formulations. Thus, principal write-down CoCo debt turns out to give the highest risk reduction possible associated with convertible debt as it maximizes the positive effect of debt reduction in the absence of equity dilution. However, it may also demand higher yield to compensate its holders for the loss of value upon conversion.

Effect	Bail-inable Debt	Write-down	Equity Convert CoCo	Impact on Risk
Debt reduction	No	Yes	Yes	Risk Reduction
Equity dilution	No	No	Yes	Risk Enhancement
CoCo dilution	No	No	Yes	Risk Reduction
Yield	Yes	Yes	Yes	Risk Enhancement

Table 1: Effects of debt instruments on bank risk choice

4 Endogenous Yield

This section solves for the endogenous yield across debt types by a numerical calibration of the results. Assessing CoCo debt pricing in terms of its contractual features is ultimately necessary for any policy conclusions. Clearly, the yield depends both on the induced endogenous risk and the relative priority of the different forms of bank debt.

Bail-inable debt

Consider possible payoffs under bail-inable debt (as shown in the upper panels of Figure 11). Under a safe choice debtholders are repaid $C(1+y)$ if there is enough funds ($v > D + Cy$). If interim leverage is low ($v < D + Cy$), then debtholders get what is left after deposits are repaid $v - (D - C)$. Under the risky choice, their payoff depends on the realization of V_2.

We showed earlier that bail-inable debt induces a safe choice when interim asset values $v \geq v_B^* = v_D^* + Cy$. Thus, debtholders expect to get safe payoff with probability $\frac{1+\delta-v_B^*}{2\delta}$, and risky payoff otherwise. Since bail-inable debt does not repay C in all states, the promised yield must be positive for debtholders to break-even.

Note that for any types of CoCos, the payoff structure of CoCo debt holders is the same as for the holders of bail-inable debt if no conversion is triggered. However, the distribution of these payoffs depends on the induced risk choice.

Write-down CoCos

Next consider the case when the banker raises C of write-down CoCos and $D-C$ in deposits. If conversion is not triggered, holders of write-down CoCos get the same payoff as bail-inable debt holders (as shown in the upper panels of Figure 11). Holders of write-down CoCo debt get nothing if conversion is triggered, independently from the strategy of the banker (see Figure 8). Two differences between bail-inable debt and write-down CoCos affect their yields. First, in the states when conversion is triggered, write-down CoCos offer no payment even if the bank is solvent, reducing the payoff to CoCo holders. Second, write-down CoCos discourage risk, thus enhancing the probability of repayment when the interim value is not revealed. The chance of getting a certain payoff equal to $\min\{C(1+y), v - D + C\}$ is higher in case of write-down

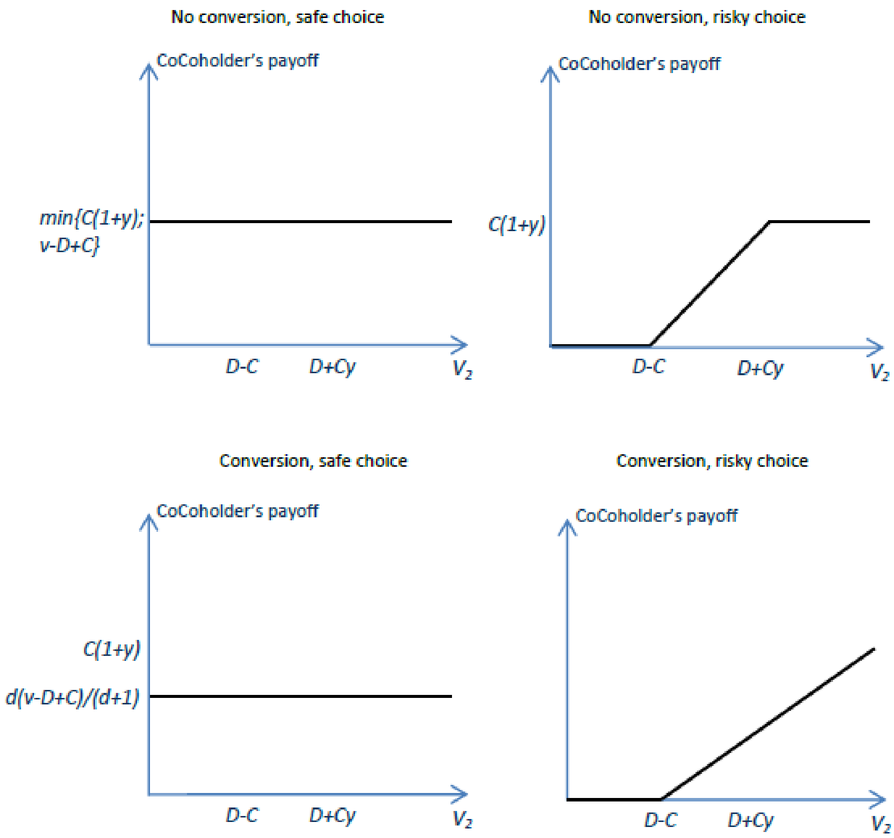

Figure 11: Payoff of CoCo debt converted to equity under different banker's choice

CoCos.

Intuitively, these depend on the trigger value v_T and its precision φ. A higher trigger as well as a higher precision increase the yield to compensate for the CoCoholders' losses in conversion. At the same time, they also induce a safer banker's choice, and thus reduce the required yield.

Equity conversion CoCo debt

We now consider the case of CoCos with a fixed equity conversion ratio.

If conversion is triggered, holders of CoCo debt receive a share of equity $\frac{d}{d+1}$. In the case of safe asset choice when $v_C^* \leq v < v_B^*$, they get a fraction of the safe payoff $\frac{d}{d+1} \cdot (v - D + C)$. In case of a risky choice when $v < v_C^*$, their payoff depends on the asset realization at $t = 2$. If the CoCos are not converted, holders of converted to equity CoCo debt get the same payoff as bail-inable debt holders under safe and risky strategies. Those payoffs are shown in Figure 11. Recall that risk control is chosen for $v \geq v_C^*$ (as shown in Figure 10).

There are two critical differences between bail-inable debt and converted to equity CoCo bonds that affect their yields. First, unlike bail-inable debt holders, CoCo holders face a debt dilution loss when converted. Second, CoCos induce better risk control, increasing the probability of some repayment either in the form of equity or debt (for $v_C^* \leq v < v_B^*$).

Comparing risk across CoCo debt types

Finally, we compare write-down CoCo and converted to equity CoCo debt. Again, there are two effects. Intuitively, write-down CoCos should be more expensive, since conversion leads to a complete loss of value. The resulting higher yield may reduce incentives to control risk (and will require a higher trigger). At the same time, write-down CoCo debt induces higher risk control than converted to equity CoCos because of the absence of any equity dilution effect.

The comparison of the payoff structure together with the endogenous risk choice is shown in Table 2[17]. The formulas for implicitly given yields of these three types of debt are given in the Appendix.

Debt type/ Range of v	$v < v_{WD}^*$	$v_{WD}^* \leq v < v_C^*$	$v_C^* \leq v < v_B^*$	$v \geq v_B^*$
Bail-inable debt	Risk/No Conv	Risk/No Conv	Risk/No Conv	Safe/No Conv
Write-down CoCo	Risk/Conv wp φ	Safe/Conv wp φ	Safe/Conv wp φ	Safe/No Conv
Equity convert CoCo	Risk/Conv wp φ	Risk/Conv wp φ	Safe/Conv wp φ	Safe/No Conv

Table 2: Payoff structure and endogenous risk of debt instruments

Next, we perform a numerical calibration. We focus on the bank's risk control choice as well as the magnitude of the yield for different forms of debt.

There are two key elements affecting the yield, aside from their contractual priority: the induced amount of endogenous risk and the resulting amount of risk bearing by the deposit insurance fund. This quantitative exercise is necessary to define the specific ranking of required yields for different types of debt a well as their relative effect on endogenous risk-shifting for the plausible set of parameter values.

First we characterize the yields for different forms of debt for the following set of parameter values: $D = 0.95, C = 0.025, \sigma = 0.035, p = 0.5, \delta = 0.05, z = 0.01$. Figure 12 illustrates yields for bail-inable debt, converted to equity CoCos and principal write-down CoCos for the range of values of bank initial equity $1 - D$ (upper left panel), risky asset volatility σ (upper right panel) and the amount of certain type of debt C (lower panel).

Quantitative result 1.

The yield for converted to equity CoCos may be higher than that of bail-inable debt if initial

[17]"Risk" indicates that the banker makes risky choice, "Safe" indicates safe choice, "Conv wp φ" means that conversion takes place with probability φ, "No Conv" means that no conversion takes place.

bank equity is high, and the amount of non-deposit debt C as well as risky asset volatility σ are low. In contrast, principal write-down CoCos require higher yield than converted to equity CoCos or bail-inable debt.

This result points out that the risk reduction effect of converted to equity CoCos may be sufficiently high to compensate the CoCoholders for the loss they face at conversion. So that converted to equity CoCo debt may become cheaper than bail-inable debt. In the case of write-down CoCos the CoCo dilution faced by their holders is so strong that it offsets the risk reduction effect write-down CoCos obtain. As a result, the write-down CoCo holders will require a higher yield than the holders of bail-inable debt and converted to equity CoCos.

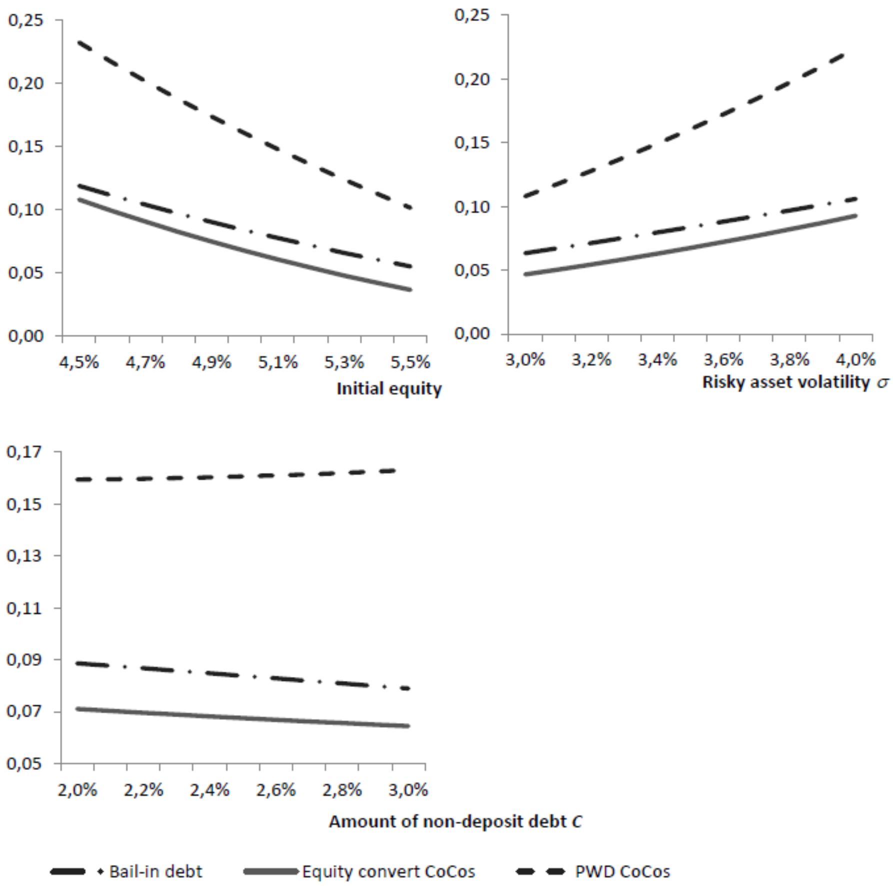

Figure 12: Yields for bail-inable debt, converted to equity and principal write-down CoCos.

Next we characterize the bank's risk control choice for different forms of debt. Figure 13 illustrates risk control thresholds for bail-inable debt v_B^*, CoCos with fixed conversion ratio v_C^* and principal write-down CoCos v_{WD}^* over the range of initial equity $1 - D$ (upper left panel), asset volatility σ (upper right panel) and the amount of debt C (lower panel).

Quantitative result 2.

A bank with principal write-down CoCos controls risk for a wider range of asset values than a bank with CoCos with a fixed conversion ratio, i.e $v_{WD}^* < v_C^*$.

Recall that in the basic model with exogenous yield we get the same result because of the absence of equity dilution effect. Here we also take into account the difference in yields for these two types of CoCos. Note from Quantitative result 1 that write-down CoCos pay a higher yield than converted to equity CoCos. In other words the yield effect (which reduces incentives to control risk) is stronger in write-down CoCos. Quantitative result 2 highlights how for our plausible set of parameter values, the yield effect is dominated by the equity dilution effect.

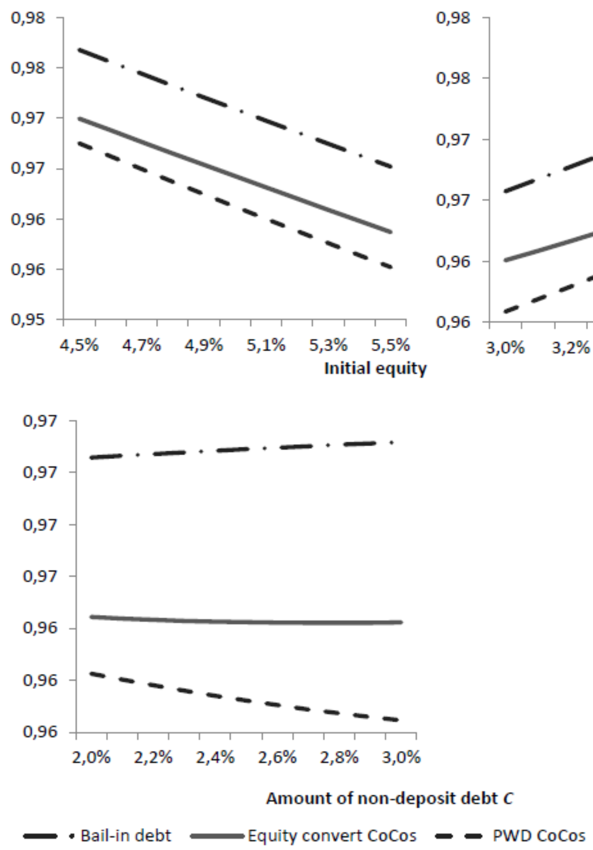

Figure 13: Risk control thresholds for bail-inable debt, converted to equity and principal write-down CoCos.

5 Conversion at maturity

In the basic model we have treated CoCos at maturity as junior bail-inable debt. This section considers an extension where CoCos convert at $t = 2$ if the asset value V_2 falls below the trigger v_T. Thus conversion can take place both as going concern at $t = 1$ as before and as gone concern at the final date $t = 2$. Conversion ratio is $d = \frac{C}{v_T - D}$.

We first characterize the risk control decision for a given yield and trigger v_T.

Consider the banker's program (similar to (8)):

$$
\begin{aligned}
&\max_e e \cdot \underbrace{[\max(v - D - Cy, 0) \cdot I(v \geq v_T) + \frac{v - D + C}{d+1} \cdot I(v < v_T)]}_{\text{equity value if } e=1} + \\
&(1-e) \cdot [\underbrace{\frac{v - z - D + C + \Delta(v + C)}{d+1} \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion at } t=1 \text{ and } e=0} + \\
&\underbrace{(F(v_T) - F(D - C)) \cdot \mathbf{E}\left(\frac{V_2 - D + C}{d+1}\Big| D - C < V_2 < v_T\right)}_{\text{equity value if no conversion at } t=1\text{, but conversion at } t=2 \text{ and } e=0} + \\
&\underbrace{(1 - F(v_T)) \cdot \mathbf{E}(V_2 - D - Cy | V_2 > v_T) \cdot [(1 - \varphi) \cdot I(v < v_T) + I(v \geq v_T)]}_{\text{equity value if no conversion at } t=1 \text{ or } t=2 \text{ and } e=0}] \\
&\text{s.t. } e \in \{0, 1\}
\end{aligned} \tag{11}
$$

where $\Delta(v+C) = -\int_{-\infty}^{D-C} (V_2 - D + C) f(V_2|v) dV_2$ is the risk-shifting incentive under CoCos conversion.

There are two major differences relative to (8). In the first place, gone concern conversion is now certain for highly levered banks with $v < v_T$ if they choose risk control. Second, less leveraged banks that take risk may now face a second chance of conversion at $t = 2$.

We next solve for the trigger that provides monotone incentives as in the basic model.

Lemma 7. *For a bank with CoCos that may convert at maturity the trigger's value* $v_T = \max[D + Cy, v_M^*]$ *ensures monotonicity of the risk control choice. Its value is higher than for CoCos that act as a junior debt at maturity* v_B^*.

Intuitively, for low levered banks the risky payoff becomes more attractive since its shareholders receive a value transfer from CoCoholders upon conversion as long as $D - C \leq V_2 < v_T$. To offset this increased risk incentive, a higher trigger is necessary.

As it is not possible to compare analytically the risk control thresholds v_M^{**} and v_C^* under an endogenous yield y, we study them under a numerical calibration.[18]

Figure 14 illustrates the risk control thresholds for both types of CoCos as a function of the amount of CoCos C. The following result also holds for the wide range of parameter values D and σ.

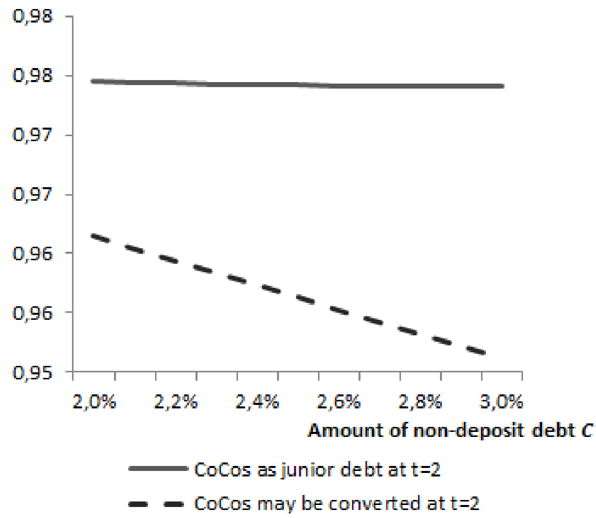

Figure 14: Risk control thresholds for CoCos with and without gone concern conversion

Quantitative result 4.

CoCos with possible conversion at maturity improve the risk control choice relative to CoCos acting as junior debt, i.e $v_M^{**} < v_C^*$*, but require a higher trigger and thus more frequent conversion.*

Intuitively, for highly levered banks with negative equity at $t = 1$ a safe choice makes conversion certain, gaining a value transfer from CoCo debt, while a risky choice reduces the chance of conversion.

6 Contingent Capital versus Equity

In this section we compare the risk reduction effect of converted to equity CoCo debt versus equity. Specifically, the question we address is what amount of contingent capital is required to provide the same risk incentives as equity?

[18] We calibrate using the following set of parameter values: $D = 0.95, C = 0.02, \sigma = 0.045, p = 0.5, \delta = 0.05, z = 0.01$. See the Appendix for the derivation of the CoCos' yield.

Suppose the bank substitutes one unit of deposits by an extra amount of equity ϵ, or by an amount $k\epsilon$ of CoCos. We solve for the level of k which guarantees an equivalent improvement in risk control as with equity. The banker's program with extra equity is:

$$\begin{aligned}&\max_{e} e \cdot \max\left[v - D + \epsilon, 0\right] + (1-e)\cdot(v - D + \epsilon - z + \Delta(v+\epsilon))\\ &\text{s.t. } e \in \{0,1\}\end{aligned} \tag{12}$$

After adding extra equity ϵ, the bank has debt $D - \epsilon$, so the amount of equity in the interim period is $v - D + \epsilon$. The bank chooses risk control whenever the difference in risky and safe asset mean return z is higher than the risk-shifting incentives $\Delta(v+\epsilon)$, or equivalently:

$$e = \begin{cases} 1 & \text{if } v \geq v_D^* - \epsilon \\ 0 & \text{if } v < v_D^* - \epsilon \end{cases} \tag{13}$$

where $v_D^* = \max[\Delta^{-1}(z), D]$ is a risk control threshold in the bank funded by deposits of amount D and equity $1 - D$[19].

The expected improvement in risk control compared to the case of deposit funding is $\frac{\epsilon}{2\delta}$, which reflects an increased range of asset values for which there is risk control. From earlier results, the improvement in risk control achieved by CoCos is $\frac{v_D^* - v_C^*}{2\delta}$, where v_C^* is the function of $k\epsilon$ instead of C.

So the condition $v_D^* - v_C^* = \epsilon$ guarantees that the expected risk control improvement from introducing extra equity ϵ and CoCos $k\epsilon$ is the same.[20]

Proposition 6. *The effect of CoCos on risk control is always weaker than of equity, unless the trigger is perfectly informative ($\varphi = 1$).*

Lemma 8. *The substitution ratio k between extra equity and CoCos k decreases in a convex way with the probability of information revelation φ if promised CoCo yield is sufficiently low. Conditions for decreasing k function are (9) is satisfied and sufficiently low yield from:*

$$\begin{aligned} y\Delta_k'[v_D^* - \epsilon(1+ky)] \quad &\geq \quad \frac{-\varphi\cdot(v_D^* - D)}{(1-\varphi)(k\epsilon + v_D^* - D)^2}\cdot\{(k\epsilon + v_D^* - D)\cdot\Delta_k'[v_D^* + \epsilon(k-1)] \\ &+ \quad (z - \Delta[v_D^* + \epsilon(k-1)])\} \end{aligned} \tag{14}$$

[19]The banker controls risk whenever $v > \Delta^{-1}(z) - \epsilon$. However, if $\Delta^{-1}(z) - \epsilon < D - \epsilon$, risk control threshold changes to $D - \epsilon$, since for negative net equity the banker never controls risk.

[20]As before, we set the trigger value to insure monotonic risk control in v, $v_T = v_B^*$.

and for its convexity - no condition (9) is satisfied and sufficiently low yield from:

$$
\begin{aligned}
y[\Delta'_k(v_D^* - \epsilon(1+ky)) + 1] \quad &\leq \quad -\frac{(v_D^* - D)}{(k\epsilon + v_D^* - D)^2} \cdot \{\Delta'_k[v_D^* + \epsilon(k-1)] \cdot (k\epsilon + v_D^* - D) \\
&+ \quad z - \Delta[v_D^* + \epsilon(k-1)]\} \qquad (15)
\end{aligned}
$$

The equivalence ratio is very sensitive to φ. As φ approaches zero, there is no amount of CoCos that can substitute equity to ensure the same risk control.

Thus a key efficiency factor for CoCos depends on the precision of the trigger to signal a state where risk-shifting incentives are high, while equity is always risk bearing. When the trigger is less precise, conversion takes less often when required. As a result, a larger amount of CoCos must be used to achieve the same risk control. However, when the required yield is sufficiently high, increasing the amount of CoCos becomes counterproductive as it implies a higher leverage and ultimately higher risk-taking.

7 Conclusions

This paper assesses how the design of bank contingent capital affects risk-taking incentives. The issue is extensively discussed in the literature, but in existing models the asset choice is exogenous (Pennacchi, 2011; Chen et al., 2013; Hilscher and Raviv, 2014). Our model identifies explicitly the effect of contractual terms of contingent capital, necessary for its optimal design and pricing. A clear result is that CoCo debt is superior to subordinated debt that may be bailed in upon default, as it actively discourages ex ante risk. This results helps to clarify the importance of going concern conversion which reduces leverage while the bank is still solvent.

The main beneficial effect of conversion is to contain risk-shifting by reducing leverage in states when risk incentives are high. We show that contrary to more traditional convertible debt (Green, 1984), equity dilution has here a negative effect on incentives. In contrast, debt dilution has a positive preventive effect. As a result, CoCo debt with a principal write-down conversion may achieve a greater reduction in risk exposure than equity conversion CoCo bonds. A countervailing effect may be caused by a higher yield, which increases effective leverage. The overall result depends on the precise terms of CoCo debt. A numerical calibration where the yield is endogenous suggests that principal write-down CoCos are more efficient in reducing risk.

The framework enables to compare the relative effectiveness of CoCos in risk reduction versus common equity, as well as other bank debt. While a one for one exchange ratio of CoCos for equity is equivalent in terms of loss absorption upon default, this is no true for its preventive effect. Contingent capital is much less efficient than equity because of limited trigger precision, which does not ensure recapitalization in all states of excessive leverage.

In future research, we plan to study how using an accounting trigger may result in regulatory forbearance, since bank reporting typically involves regulatory oversight. We are also interested in how to design CoCo debt under a constrained amount of risk absorption capacity. In general, research is needed to understand better the effect of CoCos on share pricing, which is distorted by risk-shifting.

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8 Appendix

Papers studying effect of CoCos on bank risk-taking incentives

Paper	Main assumptions	Result
Pennacchi (2011)	1. Trigger: capital ratio based on market value 2. Accurate observability of capital ratio. 3. Default can occur whenever the asset value is below deposits 4. Probability of conversion depends on asset risk	1. High conversion ratio reduces risk-taking incentives 2. CoCos induce less risk-taking than subordinated debt.
Koziol and Lawrenz (2012)	1. Trigger: cash flow 2. Accurate observability of cash flow. 3. Default can occur whenever the asset value is below deposits 4. Probability of conversion depends on asset risk	1. CoCos induce more risk-taking than straight bonds
Hilscher and Raviv (2014)	1. Trigger: asset value 2. Accurate observability of asset value. 3. Default can occur whenever the asset value is below deposits 4. Probability of conversion depends on asset risk	1. High conversion ratio reduces risk-taking incentives 2. With higher leverage and risk, CoCos may induce less risk-taking more than equity and subordinated debt.
Berg and Kaserer (2015)	1. Trigger: asset value 2. Regulator perfectly observes asset value, whereas shareholders and debtholders – only with noise 3. Default can occur only at maturity 4. Probability of conversion depends on asset risk	1. High conversion ratio reduces risk-taking incentives 2. Principal write-down CoCos induce more risk-taking than subordinated debt.
Chen et al. (2016)	1. Trigger: asset value 2. Accurate observability of asset value. 3. Endogenous choice of the time of default 4. Probability of conversion depends on asset risk	1. Conversion ratio does not affect the risk-taking if the trigger is set sufficiently high 2. CoCos can induce lower risk-taking than subordinated debt, but also to increase exposure to tail risk.
Chan and Wijnbergen (2016)	1. Trigger: asset value 2. Asset value is observed with noise 3. Default can occur whenever the asset value is below deposits 4. Asset risk affects probability of conversion and bankruptcy cost generating a trade-off for the bank	1. High conversion ratio reduces risk-taking incentives 2. Non-dilutive CoCos induce more risk-taking than subordinated debt, which in turn induces higher risk-taking than dilutive CoCos.
Our paper	1. Trigger: asset value 2. Asset value is only observed by bank shareholders, but may be revealed to the public 3. Default can occur only at maturity 4. Probability of conversion is independent of risk choice 5. Endogenous choice of the asset risk	1. High conversion ratio increases risk-taking incentives 2. CoCos induce better risk control than subordinated debt and worse than equity.

Lemma 1

We consider two distributions of V_2: normal and uniform. Denote $x = V_2 - D$. we consider x to be normally distributed with mean is $v - D - z$ and variance σ^2 as well as uniformly distributed with support $[v - D - z - \sigma\sqrt{3}, v - D - z + \sigma\sqrt{3}]$. We assume that $v - D - z + \sigma\sqrt{3} \geq 0$, else risky asset is never chosen. Moreover, $v - D - z - \sigma\sqrt{3} \leq 0$, else no risk-shifting takes place.

For a normal distribution, the expected value of bank equity (as defined in (1)) is:

$$\int_0^\infty x \cdot \frac{1}{\sigma} \cdot \phi(\frac{x - (v - D - z)}{\sigma})dx = (v - D - z) \cdot \Phi\left(\frac{v - D - z}{\sigma}\right) + \sigma \cdot \phi\left(\frac{(v - D - z)}{\sigma}\right)$$

For a uniform distribution (1) is:

$$\int_0^\infty x \cdot \frac{1}{2\sigma\sqrt{3}}dx = \frac{(v - D - z + \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}$$

Then the risk-shifting incentive $\Delta(v)$ (as defined in (2)) is for a normal distribution:

$$\begin{aligned}\Delta(v) = \quad & (1 - F(0, v)) \cdot \mathbf{E}(x|x > 0, v) - (v - D - z) = \\ & (v - D - z) \cdot \left[\Phi\left(\frac{v - D - z}{\sigma}\right) - 1\right] + \sigma \cdot \phi\left(\frac{(v - D - z)}{\sigma}\right)\end{aligned}$$

whereas for a uniform distribution:

$$\Delta(v) = (1 - F(0, v)) \cdot \mathbf{E}(x|x > 0, v) - (v - D - z) = \frac{(v - D - z - \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}$$

It is easy to show that under these distributions $\frac{\partial\Delta(v)}{\partial v}$ is non-positive. For a normal distribution it is:

$$\frac{\partial\Delta(v)}{\partial v} = \quad \Phi\left(\frac{v - D - z}{\sigma}\right) - 1 \leq 0$$

Note also that $\frac{\partial\Delta(v)}{\partial v} \geq -1$, since $\Phi\left(\frac{v-D-z}{\sigma}\right) \in [0, 1]$. For a uniform distribution:

$$\frac{\partial\Delta(v)}{\partial v} = \frac{2(v - D - z - \sigma\sqrt{3})}{4\sigma\sqrt{3}} \leq 0$$

Note also that $\frac{\partial\Delta(v)}{\partial v} \geq -1$, since $2(v - D - z + \sigma\sqrt{3}) > 0$.

Consider $\frac{\partial^2 \Delta(v)}{\partial v^2}$ for a normal distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \quad \phi\left(\frac{(v-D-z)}{\sigma}\right) \cdot \frac{1}{\sigma} \geq 0$$

For a uniform distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \frac{1}{2\sigma\sqrt{3}} \geq 0$$

Thus, risk-shifting incentives is a convex function of v.

Next, we look at $\frac{\partial \Delta(v)}{\partial \sigma}$ for a normal distribution it is:

$$\frac{\partial \Delta(v)}{\partial \sigma} = \phi\left(\frac{(v-D-z)}{\sigma}\right) \geq 0$$

For a uniform distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = -\frac{(v-D-z-\sigma\sqrt{3}) \cdot (v-D-z+\sigma\sqrt{3})}{4\sqrt{3} \cdot \sigma^2} \geq 0$$

Finally, we compute $\frac{\partial \Delta(v)}{\partial z}$ for a normal distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\left[\Phi\left(\frac{v-D-z}{\sigma}\right) - 1\right] \geq 0$$

For a uniform distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\frac{2(v-D-z-\sigma\sqrt{3})}{4\sigma\sqrt{3}} \geq 0$$

Lemma 2

First, we show that the banker with $v > v_D^*$ controls risk. When solving the problem (5), there are two cases: (1) $v > D$ (positive equity upon safe choice); (2) $v \leq D$.

If $v > D$, $e = 1$ if $z > \Delta(v)$. According to the Assumption 1, $\Delta(v)$ is decreasing in v. Then $\Delta(v) \leq z$ implies that $e = 1$ if $v \geq \Delta^{-1}(z)$, which is binding if $\Delta^{-1}(z) > D$. If $v \leq D$, for any v $e = 0$.

Thus, the solution to (5) depends on the relationship between D and $\Delta^{-1}(z)$. If $\Delta(D) > z$, $\Delta^{-1}(z) > D$ and the banker controls risk for $v > \Delta^{-1}(z)$. Otherwise, if $\Delta(D) \leq z$, $\Delta^{-1}(z) \leq$

D and the banker controls risk for $v \geq D$. Thus, $e = 1$ if $v > \max[\Delta^{-1}(z), D] \equiv v_D^*$.

Next, we show that if $\Delta(D) > z$ (implying that $\Delta^{-1}(z) > D$), the probability of risk control ($\frac{1+\delta-\Delta^{-1}(z)}{2\delta}$) decreases with σ. Note that $\frac{1+\delta-\Delta^{-1}(z)}{2\delta}$ decreases in $\Delta^{-1}(z)$. Since $\Delta^{-1}(z)$ is given by $\Delta(v) = z$, define $H(v, z, \sigma) \equiv \Delta(v) - z = 0$. We use the implicit function theorem to compute $\frac{v}{\partial\sigma}$.

$$\frac{\partial v}{\partial \sigma} = -\frac{\partial H/\partial \sigma}{\partial H/\partial v} \geq 0$$

where

$$\begin{aligned} \partial H/\partial \sigma &= \Delta'_\sigma(v) \geq 0 \\ \partial H/\partial v &= \Delta'_v(v) \leq 0 \end{aligned}$$

Proposition 1

First, we show that the banker with $v > v_D^* + Cy$ controls risk. When solving the problem (7), consider two cases: (1) $v > D + Cy$ (positive equity upon safe choice); (2) $v \leq D + Cy$.

If $v > D + Cy$, $e = 1$ if $z > \Delta(v - Cy)$. According to the Assumption 1, $e = 1$ if $v \geq \Delta^{-1}(z) + Cy$, which is binding if $\Delta^{-1}(z) > D$. If $v \leq D + Cy$, $e = 0$ for any v from this interval. Thus, the banker controls risk $e = 1$ if $v > \max[\Delta^{-1}(z), D] + Cy = v_D^* + Cy \equiv v_B^*$.

Next we show that the probability of risk control for $\Delta(D) > z$ (implying that $\Delta^{-1}(z) > D$) ($\frac{1+\delta-\Delta^{-1}(z)-Cy}{2\delta}$) decreases with σ. The proof of the of the derivative with respect to σ is identical to the one from Lemma 2.

Finally we show that $\frac{1+\delta-v_B^*}{2\delta}$ declines with C and y. Note that the derivatives of $\frac{1+\delta-v_D^*-Cy}{2\delta}$ with respect to C and y are equal to $-\frac{y}{2\delta}$ and $-\frac{C}{2\delta}$ accordingly and both are negative.

Proposition 2

When solving the problem (8), there are two cases: (1) $v > D + Cy$ (positive equity upon safe choice); (2) $v \leq D + Cy$.

If $v > D + Cy$, the first order condition to (8) is that $e = 1$ if:

$$F = \varphi \cdot \frac{z - \Delta(v + C)}{d + 1} + (1 - \varphi)[z - \Delta(v - Cy)] \geq 0 \tag{16}$$

Note that $F(v)$ is increasing in v:

$$\frac{\partial F(v)}{\partial v} = -(1-\varphi)\Delta'(v-Cy) - \varphi\frac{\Delta'(v+C)}{d+1} > 0 \tag{17}$$

Then (16) is binding if for $v = D + Cy$ it does not hold, i.e if (9) is satisfied. (9) implies that at $v = D + Cy$, there is no risk control.

Next, consider the case when (9) is not satisfied. If $v \leq D + Cy$, the first order condition to (8) is that $e = 1$ if:

$$G = \varphi \cdot \frac{z - \Delta(v+C)}{d+1} + (1-\varphi)[z - \Delta(v-Cy) - (v - D - Cy)] \geq 0 \tag{18}$$

Note that it is not clear whether $G(v)$ is increasing in v:

$$\frac{\partial G(v)}{\partial v} = -(1-\varphi)[\Delta'(v-Cy) + 1] - \varphi\frac{\Delta'(v+C)}{d+1} \tag{19}$$

where the first item is negative, and second is positive. Also note that the second derivative of $G(v)$ with respect to v is negative:

$$\frac{\partial^2 G(v)}{\partial v^2} = -(1-\varphi)\Delta''(v-Cy) - \varphi\frac{\Delta''(v+C)}{d+1} < 0 \tag{20}$$

Note however, that if $\frac{\partial G(v)}{\partial v}$ is non-positive for some $v \leq D + Cy$, it is not possible that for $v = D + Cy$ there is a risk control. Thus, if (9) is not satisfied, $G(v)$ is increasing in v. The banker controls risk if v is sufficiently high.

Thus, the solution to (8) depends on (9) and can be presented by v_C^* defined in (10). For $v_C^* \leq v \leq v_T$, the banker controls risk.

Lemma 3

Using implicit function theorem for (10), we demonstrate that higher φ reduces v_C^*, or decreases equivalently risk control:

$$\frac{\partial v_C^*}{\partial \varphi} = \begin{cases} -\frac{\partial F/\partial \varphi}{\partial F/\partial v} & \text{if (9) holds} \\ -\frac{\partial G/\partial \varphi}{\partial G/\partial v} & \text{otherwise} \end{cases} \tag{21}$$

where $\frac{\partial F}{\partial v} > 0$ and $\frac{\partial G}{\partial v} > 0$ from Lemma 2, and

$$\frac{\partial F}{\partial \varphi} = \frac{z - \Delta(v + C)}{d + 1} - [z - \Delta(v - Cy)] \geq 0 \tag{22}$$

$$\frac{\partial G}{\partial \varphi} = \frac{z - \Delta(v + C)}{d + 1} - [z - \Delta(v - Cy)] + (v - D - Cy) \geq 0 \tag{23}$$

the first item in (22) and (23) is non-negative, and the second is non-positive due to the fact that $\Delta(v - Cy) > \Delta(v + C)$ (recall that $\Delta'_v(v) \leq 0$). For $F(v) = 0$ and $G(v) = 0$, it must be that $z - \Delta(v - Cy) \leq 0$ and $z - \Delta(v + C) \geq 0$.

Thus, independent of condition (9), $\frac{\partial v_C^*}{\partial \varphi} < 0$, and trigger precision reduces v_C^*.

Lemma 4

We show that 1) $v_C^* \leq v_B^*$ (CoCos induce safer choices that bail-inable debt), and 2) it can be that $v_C^* > v_D^*$ (CoCos may induce riskier choice than deposit funding).

First, note that v_C^* decreases in φ from Lemma 3. Thus, v_C^* is at maximum when $\varphi = 0$, corresponding to:

$$\begin{cases} F(v)_{\varphi=0} = z - \Delta(v - Cy) = 0 & \text{if (9) holds} \\ G(v)_{\varphi=0} = z - \Delta(v - Cy) - (v - D - Cy) = 0 & \text{otherwise} \end{cases} \tag{24}$$

If (9) holds, $v_C^* = \Delta^{-1}(z) + Cy$, otherwise $v_C^* \leq D + Cy$ (as shown in Lemma 2). Thus, for both cases, $v_C^* \leq v_B^*$.

Second, since $v_B^* \geq v_D^*$ for any positive y, for sufficiently low φ, $v_C^* > v_D^*$ (the maximum v_C^* is v_B^*). This implies that deposit funding may provide better risk control than CoCos due to the presence of yield effect.

Lemma 5

Recall that the trigger is equal to $v_T = v_B^* = v_D^* + Cy$. Note that if the amount of CoCos increases, the trigger must go up:

$$\frac{\partial(v_D^* + Cy)}{\partial C} = y > 0 \tag{25}$$

The same holds for the increase in exogenous yield y:

$$\frac{\partial(v_D^* + Cy)}{\partial y} = C > 0 \tag{26}$$

Proposition 3

The banker's program is:

$$\begin{aligned}
&\max_e e \cdot [\underbrace{\max(v - D - Cy, 0) \cdot (I(v \geq v_T) + (1-\varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=1} + \\
&\underbrace{v - D + C \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=1}] + \\
&(1-e) \cdot [\underbrace{(v - z - D - Cy + \Delta(v - Cy)) \cdot (I(v \geq v_T) + (1-\varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=0} + \\
&\underbrace{v - z - D + C + \Delta(v + C) \cdot \varphi \cdot I(v < v_T)}_{\text{equity value if conversion and } e=0}] \\
&\text{s.t. } e \in \{0, 1\}
\end{aligned} \tag{27}$$

The proof is similar to the one of Lemma 2. This is just a special case when CoCos have conversion ratio $d = 0$. The critical value v_{WD}^* depends on condition (28) which describes when v_{WD}^* is above $D + Cy$:

$$\varphi[z - \Delta(D + C(1+y))] + (1-\varphi)(z - \Delta(D)) < 0 \tag{28}$$

v_{WD}^* is defined implicitly in (29):

$$\begin{cases} \varphi[z - \Delta(v + C)] + (1-\varphi)[z - \Delta(v - Cy)] = 0 & \text{if (28) holds} \\ \varphi[z - \Delta(v + C)] + (1-\varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \tag{29}$$

This allows us to disentangle the effect of CoCos on risk control when there is no conversion when trigger is breached.

To show this effect, we assume that introduction of CoCos only affects leverage upon conversion. Using implicit function theorem for (10), we demonstrate the direct debt reduction effect on v_C^*:

$$\frac{\partial v_C^*}{\partial C}|_{d=fixed,Cy=fixed} = \begin{cases} -\frac{\partial F/\partial C}{\partial F/\partial v}|_{d=fixed,Cy=fixed} & \text{if (9) holds} \\ -\frac{\partial G/\partial C}{\partial G/\partial v}|_{d=fixed,Cy=fixed} & \text{otherwise} \end{cases} \tag{30}$$

where

$$\frac{\partial F}{\partial v} = -\frac{\varphi}{d+1} \cdot \Delta_v'(v+C) - (1-\varphi)\Delta_v'(v-Cy) \geq 0 \tag{31}$$

$$\frac{\partial F}{\partial C}|_{d=fixed,Cy=fixed} = -\frac{\varphi}{d+1} \cdot \Delta_C'(v+C) \geq 0 \tag{32}$$

$$\frac{\partial G}{\partial v} = -\frac{\varphi}{d+1} \cdot \Delta_v'(v+C) - (1-\varphi)[\Delta_v'(v-Cy)+1] \geq 0 \tag{33}$$

$$\frac{\partial G}{\partial C}|_{d=fixed,Cy=fixed} = -\frac{\varphi}{d+1} \cdot \Delta_C'(v+C) \geq 0 \tag{34}$$

$$\tag{35}$$

Independent on (9), $\frac{\partial v_C^*}{\partial C}|_{d=fixed,Cy=fixed} \leq 0$ implying that higher C reduces leverage and decreases v_C^*. We define debt reduction effect as the risk control improvement associated with the change in leverage only (but not affected by the yield or conversion ratio) compared to the baseline case of only deposit funding $\frac{v_D^* - v_{WD}^*|_{Cy=0}}{2\delta}$, where $v_{WD}^*|_{Cy=0}$ is defined by (29) for a paid yield $Cy = 0$.

Proposition 4

The proof is similar to the one of Lemma 2. This is a special case when CoCos have conversion ratio $d = \frac{C}{v-D}$. The critical value v_P^* depends on the condition (36) which describes when v_P^* is above $D + Cy$:

$$\varphi \cdot \frac{(z - \Delta[D + C(1+y)](v-D)}{v-D+C} + (1-\varphi)(z - \Delta(D)) < 0 \tag{36}$$

v_P^* is defined implicitly in (37):

$$\begin{cases} \varphi \cdot \frac{[z-\Delta(v+C)](v-D)}{v-D+C} + (1-\varphi)[z - \Delta(v-Cy)] = 0 & \text{if (36) holds} \\ \varphi \cdot \frac{[z-\Delta(v+C)](v-D)}{v-D+C} + (1-\varphi)[z - \Delta(v-Cy) - (v-D-Cy)] = 0 & \text{otherwise} \end{cases} \tag{37}$$

Note that both functions $F(v)$ and $G(v)$ are increasing in v given conversion at par:

$$\frac{\partial F}{\partial v} = \frac{\partial F}{\partial v}|_{d=fixed} + \frac{\partial F}{\partial d}\cdot\frac{\partial d}{\partial v} = \frac{\partial F}{\partial v}|_{d=fixed} + \frac{\varphi(z-\Delta(v+C))}{(d+1)^2}\cdot\frac{C}{(v-D)^2} \quad (38)$$

$$\frac{\partial G}{\partial v} = \frac{\partial G}{\partial v}|_{d=fixed} + \frac{\partial G}{\partial d}\cdot\frac{\partial d}{\partial v} = \frac{\partial G}{\partial v}|_{d=fixed} + \frac{\varphi(z-\Delta(v+C))}{(d+1)^2}\cdot\frac{C}{(v-D)^2} \quad (39)$$

$$(40)$$

where $\frac{\partial F}{\partial v}|_{d=fixed}$ and $\frac{\partial F}{\partial v}|_{d=fixed}$ are given in (17) and (19) accordingly, and $z > \Delta(v+C)$ for any $v_D^* - C \le v_C^* \le v_D^* + Cy$.

To disentangle the effect of equity dilution, we assume that introduction of CoCos only affects conversion ratio with leverage and yield being fixed. Using implicit function theorem for (10), we show the equity dilution effect on v_C^*:

$$\frac{\partial v_C^*}{\partial d}|_{C=fixed,Cy=fixed} = \begin{cases} -\frac{\partial F/\partial d}{\partial F/\partial v}|_{C=fixed,Cy=fixed} & \text{if (9) holds} \\ -\frac{\partial G/\partial d}{\partial G/\partial v}|_{C=fixed,Cy=fixed} & \text{otherwise} \end{cases} \quad (41)$$

where $\frac{\partial F}{\partial v}|_{C=fixed,Cy=fixed}$ and $\frac{\partial G}{\partial v}|_{C=fixed,Cy=fixed}$ are given as in (31) and (33), and

$$\frac{\partial F}{\partial d}|_{C=fixed,Cy=fixed} = -\frac{\varphi(z-\Delta(v+C))}{(d+1)^2} \le 0 \quad (42)$$

$$\frac{\partial G}{\partial d}|_{C=fixed,Cy=fixed} = -\frac{\varphi(z-\Delta(v+C))}{(d+1)^2}) \le 0 \quad (43)$$

$$(44)$$

Independent on (9), $\frac{\partial v_C^*}{\partial d}|_{C=fixed,Cy=fixed} \ge 0$ implying that the increase in d through higher amount of CoCos increases the dilution of the banker's share and increases v_C^*. We define equity dilution effect as the reduction in risk control associated with the change in conversion ratio compared to the case of write-down CoCos with no conversion into shares $\frac{v_{WD}^*|_{Cy=0} - v_P^*}{2\delta}$, where $v_P^*|_{Cy=0}$ is defined by (37) for a paid yield $Cy = 0$.

Since higher conversion ratio implies higher risk control, $v_P^* > v_{WD}^*$.

Proposition 5

The proof is based on the one of Lemma 2 for $d = \frac{C}{v_T - D}$. The critical value v_C^* depends on (45) which describes when v_C^* is above $D + Cy$:

$$\varphi \cdot \frac{(z - \Delta[D + C(1+y)](v_T - D)}{v_T - D + C} + (1-\varphi)(z - \Delta(D)) < 0 \tag{45}$$

v_C^* is defined implicitly in (46):

$$\begin{cases} \varphi \cdot \frac{[z-\Delta(v+C)](v_T-D)}{v_T-D+C} + (1-\varphi)[z - \Delta(v - Cy)] = 0 & \text{if (45) holds} \\ \varphi \cdot \frac{[z-\Delta(v+C)](v_T-D)}{v_T-D+C} + (1-\varphi)[z - \Delta(v - Cy) - (v - D - Cy)] = 0 & \text{otherwise} \end{cases} \tag{46}$$

Note that fixed conversion $d = \frac{C}{v_T - D}$ produces lower equity dilution to the banker than conversion at par with $d = \frac{C}{v - D}$, since conversion occurs only for $v < v_T$. Recall from Proposition 4 that lower equity dilution results in better risk control. This implies that changing conversion ratio from at par to a fixed one produces better risk control. As a result, $v_C^* < v_P^*$.

Changing conversion from at par to the fixed one increases the ex ante probability of bank risk control $\frac{1+\delta-v_C^*}{2\delta}$. We call this increase a CoCo dilution effect as the risk control increase associated with the lower equity dilution and value transfer from CoCoholders to the equityholders (banker). We denote it by $v_C^*|_{Cy=0} - v_P^*|_{Cy=0}$, where $v_C^*|_{Cy=0}$ is defined by (46) for a paid yield $Cy = 0$.

Lemma 6

Note that fixed conversion ratio $d = \frac{C}{v_T - D}$ decreases with v_T. Thus, if v_T goes up, conversion ratio goes down. From Proposition 5, lower d increases risk control by diluting CoCoholders. As a result, higher v_T enhances CoCos' risk reduction effect.

Endogenous yield

Here we give the implicit conditions for yield from break-even conditions of holders of different forms of debt.

Bail-inable debt

$$C = \frac{1+\delta - v_B^*}{2\delta} \cdot C(1+y) + \frac{v_B^* - 1 + \delta}{2\delta} \cdot \Big[Prob(V_2 \geq D + Cy | v < v_B^*) \cdot C(1+y) \tag{47}$$
$$+ Prob(D - C \leq V_2 < D + Cy | v < v_B^*) \cdot \mathbf{E}(V_2 - D + C | D - C \leq V_2 < D + Cy, v < v_B^*) \Big].$$

Write-down CoCos

$$C = \frac{1+\delta - v_B^*}{2\delta} \cdot C(1+y) + \frac{v_B^* - v_{WD}^*}{2\delta} \cdot (1-\varphi)C(1+y) +$$
$$\frac{v_{WD}^* - 1 + \delta}{2\delta} \cdot (1-\varphi)\Big[Prob(V_2 \geq D + Cy | v < v_{WD}^*) \cdot C(1+y) + \tag{48}$$
$$Prob(D - C \leq V_2 < D + Cy | v < v_{WD}^*) \cdot \mathbf{E}(V_2 - D + C | D - C \leq V_2 < D + Cy, v < v_{WD}^*)\Big].$$

Equity conversion CoCos

$$C = \frac{1+\delta - v_B^*}{2\delta} \cdot C(1+y) + \frac{v_B^* - v_C^*}{2\delta} \cdot \Big((1-\varphi)C(1+y) + \varphi \cdot \frac{d}{d+1} \cdot (v - D + C)\Big)$$
$$\frac{v_C^* - 1 + \delta}{2\delta} \cdot \Big[(1-\varphi)\Big(Prob(V_2 \geq D + Cy | v < v_C^*) \cdot C(1+y) + \tag{49}$$
$$Prob(D - C \leq V_2 < D + Cy | v < v_C^*) \cdot \mathbf{E}(V_2 - D + C | D - C \leq V_2 < D + Cy, v < v_C^*)\Big) +$$
$$\varphi \cdot Prob(V_2 > D - C | v < v_C^*) \cdot \frac{d}{d+1} \cdot \mathbf{E}(V_2 - D + C | V_2 > D - C, v < v_C^*)\Big].$$

Next we define the yield of CoCo debt that may convert at maturity.

CoCos that may convert at $t = 2$

$$C = \frac{1+\delta - v_M^{**}}{2\delta} \cdot C(1+y) + \frac{v_M^{**} - 1 + \delta}{2\delta} \cdot \Big[(1 - F(v_T^M)) \cdot C(1+y) +$$
$$(F(v_T^M) - F(D - C)) \cdot \frac{d}{d+1} \cdot \mathbf{E}(V_2 - D + C | D - C < V_2 < v_T^M)\Big] \tag{50}$$

where v_T^M is a properly set trigger given implicitly by $v_T = \max[D + Cy, v_M^*]$.

Proof of Lemma 7

The solution to the problem (11) identifies two new critical interim asset values. First, v_M^* is the risk control threshold for those banks with asset value above the trigger, given as a maximum of $D + Cy$ and the asset value given implicitly by:

$$\Pi_{nc}^R - v - D - Cy = 0 \tag{51}$$

where

$$\Pi_{nc}^R = \quad (1-\varphi)\cdot[(F(v_T)-F(D-C))\cdot\mathbf{E}\left(\frac{V_2-D+C}{d+1}\Big|D-C<V_2<v_T\right)+$$
$$(1-F(v_T))\cdot\mathbf{E}(V_2-D-Cy|V_2>v_T)] \tag{52}$$

is the bank's payoff under no risk control and no conversion at $t=1$.

Second, v_M^{**} is the risk control threshold for banks with asset value below v_T, given by :

$$\Pi_{nc}^R+\varphi\cdot\frac{v-z-D+C+\Delta(v+C)}{d+1}-\frac{v-D+C}{d+1}=0 \tag{53}$$

We observe that bank's risk control choice is not monotone in its asset value v: banks with $v\in[v_T,v_B^*)$ still do not control risk (as without CoCos), whereas highly levered banks with asset value $v\in[v_M^{**},v_T)$ choose to exercise risk control. The introduction of CoCos with conversion at maturity implies that banks with $v\in(v_B^*,v_M^*]$ give up on risk control.

Next we show that $v_M^*\geq v_B^*$. Recall that v_B^* is implicitly given by $\max(v-D-Cy,0)=v-z-D-Cy+\Delta(v-Cy)$ from (7), whereas v_M^* is implicitly given by $\max(v-D-Cy,0)=\Pi_{nc}^R$ from (11) and (52).

If $\Pi_{nc}^R\geq v-z-D-Cy+\Delta(v-Cy)$, then the risky payoff is higher for CoCos with conversion at $t=2$, and it must be that $v_M^*\geq v_B^*$. The difference between those payoffs $\Pi_{nc}^R-v-z-D-Cy+\Delta(v-Cy)$ equals to:

$$(F(v_T)-F(D+Cy))\cdot\Big[\mathbf{E}\Big(\frac{V_2-D+C}{d+1}\Big|D+Cy\leq V_2<v_T\Big)-$$
$$\mathbf{E}(V_2-D-Cy|D+Cy\leq V_2<v_T)\Big]+$$
$$(F(D+Cy)-F(D-C))\cdot\mathbf{E}\left(\frac{V_2-D+C}{d+1}\Big|D-C\leq V_2<D+Cy\right) \tag{54}$$

The item in the last line is positive.The item in the first two lines is non-negative as soon as $\frac{V_2-D+C}{d+1}>V_2-D-Cy$ for the range $D+Cy\leq V_2<v_T$. It is enough to show that

$$\frac{V_2-D+C}{d+1}>V_2-D \tag{55}$$

which for $d=\frac{C}{v_T-D}$ is

$$\frac{(V_2-D+C)(v_T-D)}{v_T-D+C}-\frac{(V_2-D)(v_T-D+C)}{v_T-D+C}=\frac{C(v_T-V_2)}{v_T-D+C} \tag{56}$$

which is non-negative for $V_2 < v_T$. This implies that $v_M^* \geq v_B^*$ (equality may occur if both thresholds are equal to $D + Cy$).

Proposition 6

To compute the substitution ratio k, we use (10) given $v_C^* = v_D^* - \epsilon$ and $C = k\epsilon$:

$$\begin{cases} F(v_D^* - \epsilon | k\epsilon, d) = 0 & \text{if (9) holds} \\ G(v_D^* - \epsilon | k\epsilon, d) = 0 & \text{otherwise} \end{cases} \tag{57}$$

or equivalently

$$\begin{cases} \frac{\varphi}{d+1} \cdot (z - \Delta[v_D^* + \epsilon(k-1)]) + (1-\varphi) \cdot (z - \Delta[v_D^* - \epsilon(1+ky)]) = 0 \\ \frac{\varphi}{d+1} \cdot (z - \Delta[v_D^* + \epsilon(k-1)]) + (1-\varphi) \cdot (z - \Delta[v_D^* - \epsilon(1+ky)] - [v_D^* - D - \epsilon(1+ky)]) = 0 \end{cases}$$

Here we prove that $k \geq 1$. The proof is by contradiction. Assume that $k < 1$. Consider $F(v_D^* - \epsilon | k\epsilon, d) = 0$. Note that $\Delta[v_D^* - \epsilon] \geq z$, since banker with $v < v_D^*$ does not control risk by definition of v_D^*. Since the whole expression is equal to zero, and the second term is non-negative, the first term should be non-positive. Hence,

$$\Delta[v_D^* + \epsilon(k-1)] - z \leq 0$$

This is true only if $v \geq v_D^*$. If $k < 1$, then $v_D^* + \epsilon(k-1) < v_D^*$. This is a contradiction. The same logic applies to $G(v_D^* - \epsilon | k\epsilon, d) = 0$, where the second item is also negative, and even more smaller than $z - \Delta[v_D^* - \epsilon]$.

Thus, it always holds that $k \geq 1$, and higher amount of CoCos is required to provide the same effect as equity.

Note that if $\varphi = 1$, it must be the case that $z - \Delta[v_D^* + \epsilon(k-1)] = 0$, which is true only if $k = 1$.

Lemma 8

To show the effect of φ on k, we compute $\frac{\partial k}{\partial \varphi}$ and $\frac{\partial^2 k}{\partial \varphi^2}$. Rewriting (10) using $d = \frac{k\epsilon}{v_D^* - D}$, $v_C^* = v_D^* - \epsilon$ and $C = k\epsilon$ and applying implicit function theorem yields:

$$\frac{\partial k}{\partial \varphi} = \begin{cases} -\frac{\partial F(v_D^* - \epsilon|k\epsilon)/\partial \varphi}{\partial F(v_D^* - \epsilon|k\epsilon)/\partial k} & \text{if (9) holds} \\ -\frac{\partial G(v_D^* - \epsilon|k\epsilon)/\partial \varphi}{\partial G(v_D^* - \epsilon|k\epsilon)/\partial k} & \text{otherwise} \end{cases} \tag{58}$$

$\frac{\partial F(v_D^* - \epsilon|k\epsilon)}{\partial k}$ equals to

$$\begin{aligned} &\frac{\varphi \cdot (v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot \left(\underbrace{-(k\epsilon + v_D^* - D) \cdot \Delta_k'[v_D^* + \epsilon(k-1)]}_{\geq 0} - \underbrace{(z - \Delta[v_D^* + \epsilon(k-1)])}_{\geq 0} \right) \\ &+ (1-\varphi)\epsilon y \underbrace{\Delta_k'[v_D^* - \epsilon(1+ky)]}_{\leq 0} \end{aligned} \tag{59}$$

where the item in the first line is non-negative for infinitesimal ϵ. The whole expression is non-negative if the yield y is sufficiently small.

$\frac{\partial G(v_D^* - \epsilon|k\epsilon)}{\partial k}$ equals to

$$\begin{aligned} &\frac{\varphi \cdot (v_D^* - D) \cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot \left(\underbrace{-(k\epsilon + v_D^* - D) \cdot \Delta_k'[v_D^* + \epsilon(k-1)]}_{\geq 0} - \underbrace{(z - \Delta[v_D^* + \epsilon(k-1)])}_{\geq 0} \right) \\ &+ (1-\varphi)\epsilon y \underbrace{[\Delta_k'[v_D^* - \epsilon(1+ky)] + 1]}_{\geq 0} \end{aligned} \tag{60}$$

where the item in the first line is non-negative for infinitesimal ϵ, and the item in the second line is also non-negative due to Assumption 1. Derivatives with respect to φ are:

$$\begin{aligned} \frac{\partial F(v_D^* - \epsilon|k\epsilon)}{\partial \varphi} &= \underbrace{\frac{(v_D^* - D)}{k\epsilon + v_D^* - D} \cdot (z - \Delta[v_D^* + \epsilon(k-1)])}_{\geq 0} - \underbrace{(z - \Delta[v_D^* - \epsilon(1+ky)])}_{\leq 0} \geq 0 \\ \frac{\partial G(v_D^* - \epsilon|k\epsilon)}{\partial \varphi} &= \frac{\partial F(v_D^* - \epsilon|k\epsilon)}{\partial \varphi} + (v_D^* - D - \epsilon(1+ky)) \geq 0 \end{aligned} \tag{61}$$

Thus, $\frac{\partial k}{\partial \varphi} \leq 0$ if (9) is satisfied and yield y is sufficiently small, so that (14) is satisfied. Next, consider $\frac{\partial^2 k}{\partial \varphi^2}$:

$$\begin{aligned} = \quad & \frac{\frac{\partial F}{\partial \varphi} \cdot \left(\frac{(v_D^* - D)\cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot [-\Delta_k'[v_D^* + \epsilon(k-1)] \cdot (k\epsilon + v_D^* - D) - (z - \Delta[v_D^* + \epsilon(k-1)]] \right)}{\left(\frac{\partial F}{\partial k}\right)^2} \\ - \quad & \frac{\frac{\partial F}{\partial \varphi} \cdot \epsilon y \Delta_k'(v_D^* - \epsilon(1+ky))}{\left(\frac{\partial F}{\partial k}\right)^2} \geq 0 \end{aligned} \tag{62}$$

if (9) is satisfied and otherwise:

$$\begin{aligned} \frac{\partial^2 k}{\partial \varphi^2} \quad = \quad & \frac{\frac{\partial G}{\partial \varphi} \cdot \left(\frac{(v_D^* - D)\cdot \epsilon}{(k\epsilon + v_D^* - D)^2} \cdot (-\Delta_k'[v_D^* + \epsilon(k-1)] \cdot (k\epsilon + v_D^* - D) - (z - \Delta[v_D^* + \epsilon(k-1)]) \right)}{\left(\frac{\partial G}{\partial k}\right)^2} \\ - \quad & \frac{\frac{\partial G}{\partial \varphi} \cdot \epsilon y [\Delta_k'(v_D^* - \epsilon(1+ky)) + 1]}{\left(\frac{\partial G}{\partial k}\right)^2} \geq 0 \end{aligned} \tag{63}$$

which is non-negative for sufficiently low yield y.

The substitution ratio k is a convex function of φ if the promised yield y is sufficiently low, i.e when (15) is satisfied.