FINANCE

MACRO RISK

Risk and Macro Finance
Working Paper Series

No. 2016-02

Insecure Debt

Rafael Matta and Enrico Perotti

University of Amsterdam

Risk and Macro Finance is the acclaimed research focal area of the University of Amsterdam's Faculty of Economics and Business.

The Risk and Macro Finance Working Papers Series is downloadable at http://www.acrm.uva.nl

Risk and Macro Finance Working Paper Series:
[2013.1] Modelling Financial Contagion Using Mutually Exciting Jump Processes, Yacine Aït-Sahalia, Julio Cacho-Diaz and Roger J. A. Laeven, January 2013.
[2013.2] Mutual Excitation in Eurozone Sovereign CDS, Yacine Aït-Sahalia, Roger J. A. Laeven and Loriana Pelizzon, March 2013.
[2013.3] On Zombie Banks and Recessions after Systemic Banking Crises: It Does Matter How Governments Intervene, Timotej Homar and Sweder van Wijnbergen, April 2013.
[2013.4] Financial Fragility, Sovereign Default Risk and the Limits to Commercial Bank Rescues, Christiaan van der Kwaak and Sweder van Wijnbergen, October 2013.
[2014.1] Cocos, Contagion and Systemic Risk, Stephanie Chan and Sweder van Wijnbergen, August 2014.
[2014.2] Financial Fragility and the Fiscal Multiplier, Christiaan van der Kwaak and Sweder van Wijnbergen, August 2014.
[2014.3] Consumption and Portfolio Choice under Loss Aversion and Endogenous Updating of the Reference Level, Servaas van Bilsen, Roger J. A. Laeven and Theo E. Nijman, November 2014.
[2015.1] The Formation of a Core Periphery Structure in Heterogeneous Financial Networks, Daan in 't Veld, Marco van der Leij and Cars Hommes, April 2015.
[2015.2] Safe Asset Demand: A Review, Pascal Golec and Enrico Perotti, September 2015.
[2015.3] Technological Change and the Evolution of Finance, Robin Döttling and Enrico Perotti, September 2015.
[2016.1] Coco Design, Risk Shifting Incentives and Financial Fragility, Stephanie Chan and Sweder van Wijnbergen, January 2016.
[2016.2] Insecure Debt, Rafael Matta and Enrico Perotti, August 2016

Insecure Debt*

Rafael Matta
University of Amsterdam
matta@uva.nl

Enrico Perotti
University of Amsterdam and CEPR
e.c.perotti@uva.nl

This Draft: August 26, 2016

Abstract

We study bank funding choices under asset liquidity risk and a realistic bankruptcy process, where illiquid assets are shared among all unpaid creditors. Repo debt is cheap and stable but shifts risk to unsecured debt. In the unique equilibrium, repo has a nonmonotonic effect. Runs are rare when unpledged liquid assets are abundant, rise as more repo funding shifts risk, and ultimately fall as less liquidity is available for early withdrawals. The socially optimal choice minimizes inefficient runs by limiting repo or by subsidizing a high rollover yield on unsecured debt. The private choice uses more repo and a lower rollover reward, trading off runs against cheaper funding.

Key words: Repo credit, secured debt, bank runs, repo runs, liquidity risk, bankruptcy privileges.
JEL classification: D8, G21.

1 Introduction

The recent US credit expansion was boosted by strong demand for safe assets (Caballero and Krishnamurthy (2009)), satisfied by intermediaries that issued safer liabilities such as short term commercial paper as well as debt secured on financial collateral (repurchase agreements or *repo*).[1] Credit was largely directed at long term assets, increasing maturity and liquidity mismatch (Krishnamurthy and Vissing-Jorgensen (2012)). Once credit and liquidity risk became apparent, many intermediaries suffered massive outflows of unsecured debt (Brunnermeier, 2009). In contrast, repo credit demanded higher haircuts (Gorton and Metrick (2012)) but was mostly rolled over, up to the eve of Lehmann's default (Krishnamurthy, Nagel and Orlov (2012)). While unsecured lenders suffered heavy losses, repo lenders were able to repossess and sell the pledged collateral.

The experience has led to sharper scrutiny of liquidity mismatch, when illiquid long term loans are supported by short term funding. The concern is supported by some evidence that securitized asset prices fell way too low during the crisis. Figure 1 shows how market prices of tranches of mortgage-backed securities with an original rating of AAA first crashed, then rebounded to levels close to the pre-crisis period.

This paper studies the effect of secured (repo) credit on run incentives by unsecured lenders, in a context of asset liquidity risk. Intermediaries are able to raise cheaper funding from investors seeking absolute safety by the pledging of liquid collateral. This may result in an optimal insurance arrangement that assigns all risk to agents willing to bear it, and compensate them by a higher yield. Repo funding is also praised for its stability, although it may trigger sudden rises in margins and force rapid deleveraging (Gorton and Metrick (2012)). We study here the less examined direct effect of repo debt on unsecured short term debt, known to be vulnerable to inefficient runs (Diamond and Dybvig (1983)).

We adopt the standard global game setting to analyse unique run equilibria of demandable debt (Morris and Shin (2003) and Goldstein and Pauzner (2005)). In order to understand the

---

[1]Next to repo debt, secured financial credit includes margins on derivative positions.
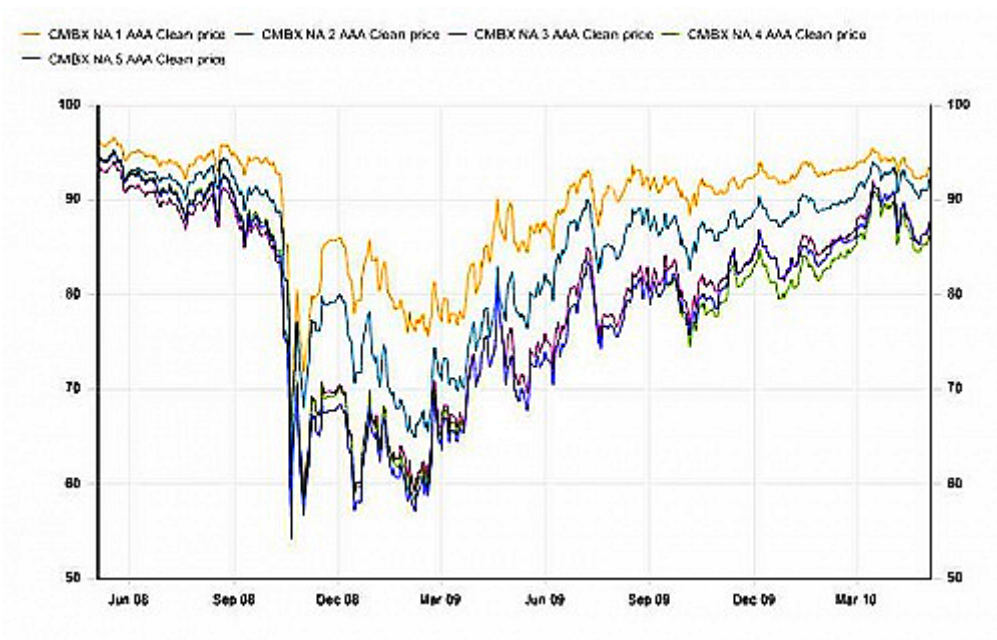
Figure 1: Markit CMBX Index Value of MBS Tranches with AAA Rating

effect of repos' claim on liquid collateral, we also model precisely how bankruptcy law treats differently liquid and illiquid assets. While in traditional bank run models all assets are sold immediately to satisfy withdrawals, in reality a bank is declared insolvent as soon as it is unable to master enough liquidity to meet withdrawals. At that point mandatory stay is triggered, with the aim to ensure orderly liquidation of assets.[2] As a result, in default illiquid assets are shared by all lenders, including those who chose to roll over. Matta and Perotti (2016) show how this feature produces a nonmonotonic effect of liquid assets on run incentives, even when fundamental risk is infinitesimal.

The model enables to show how the pledging of collateral to repo debt creates a trade-off between funding costs and the chance of runs, by adjusting both the allocation of risk and liquidity across lenders. Our key insight is that the collateral pledge has two effects. First, it reassigns safe assets, increasing the risk born by each unit of unsecured debt. Naturally, this increased risk is compensated by a higher yield. Second, the pledge also reduces the liquidity available to running unsecured lenders run.

---

[2] Bankruptcy law was introduced precisely to solve the externality created when creditors grab or liquidate assets in an uncoordinated fashion, destroying value.

We show that when repo funding is low, liquidity available to withdrawers is abundant, boosting confidence of full repayment. As secured debt rises, more liquid collateral is pledged, so unsecured lenders bear more risk and their incentive to withdraw rises, even when fundamental risk is arbitrarily low. Finally, for high levels of secured funding the appeal of running decreases, as most unpledged assets are illiquid. Because of the equal treatment imposed by mandatory stay on these assets, the relative payoff of a run relative to the rollover choice declines.[3] Thus our main result is that the use of repo debt may either improve or reduce financial stability.

We show how a social planner aiming at reducing the frequency of inefficient runs would tend to limit repo funding while offering high rollover rents on unsecured debt to discourage runs. However when asset returns are low and do not support a high rollover rent, using some repo funding can support stability in two ways. First, its low cost enables to subsidize a higher rollover yield. Second, as it subtracts liquidity available to runners it also reduces the relative run payoff relative to rollover. However, the funding choice of a private intermediary will seek to minimize funding costs net of run losses.

We show that the private choice of repo debt is always (weakly) higher, and the offered rollover yield on unsecured pricing lower than the social optimum, resulting in a higher frequency of inefficient runs. Thus another implication of our model is that unregulated secured funding can add run risk, a loss to be traded off against its lower cost.

In sum, granting greater security to some lenders is temptingly cheap, but makes other lenders more insecure. As runs create larger deposit insurance losses (both because of asset repossession as well as a higher run frequency), regulatory policy should monitor the scale of secured funding, recognize its impact on outflows of unsecured debt in liquidity stress tests and price its effect on deposit insurance losses.

The model relies on simple if natural assumptions on intermediary funding, namely the use of demandable debt, to obtain a tractable close form solution. This choice has been rationalized

---

[3]The presence of mandatory reserves ensures that the run payoff never goes to zero.

in a similar setting as the optimal instrument to satisfy contingent needs for transaction services (Stein (2012) and Matta and Perotti (2016)), a primary function of bank liabilities. This role also justifies our assumption on a regulatory minimum for liquid reserves.[4] An alternative explanation views demandable debt as a solution to insure liquidity needs as in Diamond and Dybvig (1983). In our setup, agents exposed to extreme liquidity risk would hold repo debt and accept a lower yield, while those with simple transaction needs may prefer unsecured demandable debt.[5]

Related Literature

We follow the literature in explaining repo funding in terms of a strong demand for absolute safety. A strong investor preferences for safety has now been extensively documented (Gorton and Metrick (2012) and Krishnamurthy and Vissing-Jorgensen (2012)). Recent models of instability assume a subset of infinitely risk averse agents (Caballero and Fahri (2013) and Gennaioli et al. (2013)) or a minimum bound on subsistence wealth (Ahnert and Perotti (2014)). Auh and Sundaresan (2014) argue that repo funding demands collateral to avoids violations of absolute priority,[6] while Martin et al. (2014) argue that collateral is needed when asset values may not be verifiable. While repo loans is cheaper, banks will not issue too much when collateral liquidity is low to avoid liquidation losses. Kuong (2013) shows that as unsecured debt responds to higher repo margins by demanding higher required return, the resulting higher leverage directly affects risk taking by borrowers. He and Xiong (2011) provide a dynamic model of strategic run behavior when debt is staggered. We depart from this literature by abstracting from the effect of repo on the liquidity of the pledged collateral, by assuming it is backed by liquid securities. An earlier version allowed repo haircuts to be adjusted at the interim date, producing a further amplification of the effect of repo on runs.

---

[4]An alternative explanation for very short term funding views it as a socially suboptimal choice to avoid dilution (Brunnermeier and Oehmke (2013)).

[5]In our setting it is always possible to refinance any repo withdrawal by reassigning the released liquid collateral.

[6]Violation of absolute priority may be ex post efficient, e.g. to ensure proper continuation incentives.

Our paper is to our knowledge the first to consider the interaction of repo with demandable debt under asset liquidity risk. Martin et al. (2014) and Oehmke (2014) do not compare repo with debt of the same maturity, focusing on the effect of securitization on the liquidity of pledged collateral. Gorton and Ordoñez (2014) elaborate on the insight that information-insensitive claims arise to overcome adverse selection (Pennacchi and Gorton (1999)). In such a context, collateral illiquidity will arise suddenly whenever its value becomes information sensitive. Ahnert at al. (2016) show how a higher degree of asset encumbrance due to covered bonds increases financial fragility monotonically by shifting more fundamental risk to other creditors.

The direct effect of repo we identify is quite distinct from the liquidity externality known to be associated with repo's fire sales of collateral upon default.[7] The ability of secured financial creditors to gain immediate access to pledged collateral upon default is a unique privilege. This special bankruptcy treatment (also called the "safe harbor" status) exempts repo from mandatory stay by creating a proprietary right directly enforceable on assets. This avoids risks such as excessive issuance or imperfect enforcement that may dilute a claim value. Legal scholars question whether it is justified to grant superior bankruptcy privileges to repo and derivatives (e.g. Morrison et al. (2015)). Bolton and Oehmke (2011) shows it leads to risk shifting incentives with derivatives. Duffie and Skeel (2012) argue that only cash-like collateral should be excluded from automatic stay.[8] While this limit to eligibility for safe harbor status would reduce the risk of fire sales, it would not avoid the direct effect of repo on unsecured debt risk bearing we describe. In conclusion, monitoring balance sheet encumbrance is a key stability task for banking supervisors. However, a proper welfare assessment of secured debt should take into account its role in satisfying a diffused demand for safety. The issue becomes salient in a situation of excess demand for safe assets which can impact the aggregate economy (Caballero and Fahri (2013)).

---

[7]On the financial and legal incentives to quickly resell seized collateral, see Perotti (2014) and Duffie and Skeel (2012).

[8]This is equivalent to a "narrow shadow banking model", also invoked in Gorton and Metrick (2012).

2 The Model

The economy lasts for three periods $t = 0, 1, 2$. It is populated by a bank and a continuum of lenders indexed by i. The intermediary has access to a project that needs one unit of funding in $t = 0$. Each lender is endowed with one unit. Some investors are risk neutral and demand a minimum expected return of $\gamma > 1$, reflecting their alternative storage option between $t = 0$ and $t = 2$. Other investors are infinitely risk averse, willing to lend if and only if assured to be repaid in full in all states. In exchange for absolute safety, they accept a lower return of 1.[9]. The bank raises one unit of funding by choosing a fraction s of secured funding, which can be made absolutely safe and is thus targeted to the most risk averse agents for a safe return of 1. It then raises a fraction $1 - s$ in unsecured demandable debt that promises one unit at $t = 1$ in case of early withdrawal and a rollover yield d at $t = 2$.[10] The measure of each subset is sufficiently large such that the bank could in principle finance the project with only one type of lender.

- *Project*

The value of the assets at $t = 1, 2$ is $y_t(\omega, \theta)$, where $\omega \in \{H, L\}$ is the aggregate state and $\theta \sim U\left(\underline{\theta}(\omega), \overline{\theta}\right)$. With probability λ the state is revealed to be high ($\omega = H$) and there is no fundamental risk, with the project yielding more than its opportunity cost ($y_2(H, \theta) = r > \gamma$). With probability $1 - \lambda$ the state is revealed to be low ($\omega = L$) in which case there may be fundamental and asset liquidity risk. The early liquidation value is $y_1(\omega, \theta) = k + v(\theta)$, where $k > 0$ is a safe component and $v(\theta) = \min\{\theta, c + 1 - k\}$ is an uncertain value, which can be sold at a fixed liquidation cost $c > 0$. Thus early liquidation of illiquid assets is feasible only if $v(\theta) \geq c$. We henceforth denote net liquidation revenues as $k + q(v(\theta) - c)$, where $q = 1$ for $\theta \geq c$ and $q = 0$ for $\theta < c$.

In the high state $\underline{\theta}(H) = 1 - k + c < \overline{\theta}$ so that assets are never worth less than the initial investment. In the low state the value of illiquid assets may be as low as zero ($\underline{\theta}(L) = 0$) and

---

[9]As long as secured debt is designed absolutely safe, it does not matter whether it is demandable at $t = 1$. Even if some repo lenders withdraw, the bank is able to raise more by pledging the released collateral

[10]This contract form is rationalized in a similar context by Matta and Perotti (2016) as satisfying a contingent transaction need as in Stein (2012).

there is a chance of insolvency at time 2. When asset liquidity is very low ($\theta < c$) the final asset value is only $y_2(L, \theta) = \rho < 1$. As long as c is small, the project is almost riskless when it is allowed to mature. Note that fundamental risk vanishes as c goes to zero. The project payoffs are shown in Table 1.

Table 1: Project Payoffs

$t=0$		$t=1$		$t=2$
	$\omega = H$	if held to maturity	0	r
		if early liquidation	1	0
-1	$\omega = L$	if held to maturity	0	$\begin{cases} r, & \text{if } \theta \geq c \\ \rho, & \text{if } \theta < c \end{cases}$
		if early liquidation	$k + q(v(\theta) - c)$	0

The safe portion k can be collateralized at $t = 0$, producing safe collateral that may be pledged to repo lenders or used as liquid reserve. We assume that a portion $k - \underline{k} > 0$ must be retained by the bank, so the maximum amount that can be pledged is $\underline{k}$.[11] This reserve may be rationalized as needed to meet routine withdrawals, as in Matta and Perotti (2016).

- *Lenders' Information Structure*

All agents observe the state ω at the begin of $t = 1$. In the high state the project has matured, so all claims are safe. In the low state agents receive noisy private signals on the early liquidation value of assets θ in excess of their safe component k. Their individual signal is given by

$$x_i = \theta + \sigma \eta_i, \tag{1}$$

where $\sigma > 0$ is an arbitrarily small scale parameter and η_i are i.i.d. across players and uniformly distributed over $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

---

[11]Minimum reserve requirements are a core regulatory obligation for intermediaries, now restated under the Liquidity Coverage Ratio norm of Basel III.

- *Bank Default and Orderly Liquidation*

Since all claims are safe in the high state, we focus on the low state $\omega = L$. Upon receiving their signal, lenders may choose to withdraw the principal amount 1. The bank uses its reserves sequentially to meet withdrawals. While the first creditor in the queue are ensured full repayment, once liquid reserves are exhausted the bank is forced to start a fire sale of illiquid assets. Under existing bankruptcy rules, if remaining withdrawals are larger than the net liquidation proceeds the bank is declared in default. At that point bankruptcy law forces a mandatory stay on all unsecured creditors in order to avoid the cost c and the associated fire sales, and enabling orderly resolution at $t = 2$.[12]

Under orderly liquidation, illiquid asset are worth $\ell \geq 0$, paid out to all unpaid unsecured creditors. As a result, lenders who rolled over receive a payment even in case of bank default, as it is the case in reality.

- *Lenders' Payoffs*

As repo lenders have a direct claim on the safe component k they will always be fully repaid in all states. While unsecured lenders are always repaid d when the state is high, in the low state their payoffs depend on whether they choose to roll over or withdraw and whether the bank goes bankrupt.

Let ϕ be the fraction of lenders that roll over in $t = 1$. The bank is declared bankrupt if and only if

$$(1 - \phi)(1 - s) > q(v(\theta) - c) + k - s. \tag{2}$$

Here the left hand term indicates the value demanded by unsecured lenders while the right hand side is the amount available at $t = 1$, namely the net liquidation value of illiquid assets plus the value of the unpledged collateral (which represents the payoff to repo lenders). So the bank is declared bankrupt if after paying out all reserves (unpledged safe assets) the net value of selling its nonreserve assets exceeds the claims of unpaid withdrawing depositors.

---

[12]Intuitively, meeting withdrawals by selling all assets immediately (as under strict sequential service) is in general less efficient in the case of highly illiquid assets.

Table 2: Payoffs of Unsecured Lenders in the Low State

	no bankruptcy	bankruptcy
roll over	$qd + (1-q)\frac{\rho - s - (1-s)(1-\phi)}{\phi(1-s)}$	$\frac{\ell}{1-k}$
withdraw	1	$\frac{k-s}{(1-\phi)(1-s)} + \left(1 - \frac{k-s}{(1-\phi)(1-s)}\right)\frac{\ell}{1-k}$

Unsecured lenders' payoffs in the low state are shown in Table 2. In the event of bankruptcy, the random order of arrival implies that running unsecured lenders are repaid out of the liquid reserves with probability $\frac{k-s}{(1-\phi)(1-s)}$ in $t = 1$. With probability $1 - \frac{k-s}{(1-\phi)(1-s)}$, the remaining withdrawers share the illiquid asset value under orderly liquidation with lenders who did not run in $t = 2$, each receiving $\frac{\ell}{1-k}$.

In the absence of bankruptcy, withdrawal lenders receive one unit each in $t = 1$. The remaining lenders are paid in $t = 2$, receiving d if $\theta \geq c$ or a pro rata share of the surplus $\frac{\rho - s - (1-s)(1-\phi)}{\phi(1-s)}$ if $\theta < c$.

We finally assume that: (1) the project has positive NPV even if always liquidated under orderly resolution in the low state; (2) the continuation value is higher than the liquidation value for $\theta \geq c$; and (3) the value produced under orderly liquidation is not enough to fully repay all lenders ($\ell + k < 1$), and is sufficiently low relative to the asset value r. These conditions may be stated as:

$$\lambda(r - k - \ell) - (\gamma - 1) \geq 1 - \ell - k \geq r - 1 > \bar{\theta} - c + k - 1.$$

3   Runs

For repo lenders to enjoy absolute safety, each of them requires one unit of safe collateral to ensure full repayment in all states. This implies an upper bound on total secured debt $s \leq \underline{k}$. Under this condition, repo lenders never wish to run at $t = 1$.

Consider now the unsecured lenders' rollover decision. Let $\Pi_U^R(\phi, \theta)$ be the net payoff of

unsecured lenders who roll over relative to the payoff to withdraw. We have

$$\Pi_U^R(\phi, \theta) = \begin{cases} qd + (1 - q)\frac{\rho - s - (1-s)(1-\phi)}{\phi(1-s)} - 1, & \text{if no bankruptcy} \\ -\frac{k-s}{(1-\phi)(1-s)}\left(1 - \frac{\ell}{1-k}\right), & \text{if bankruptcy} \end{cases} \tag{3}$$

Following the classic solution to global games we focus on strategic uncertainty as signals become nearly precise, that is as $\sigma \to 0$,. At the equilibrium cutoff, the threshold type must be indifferent between rolling over and withdrawing given uniform beliefs about ϕ. That is, it is the unique θ^* that solves $\int_0^1 \Pi_U^R(\phi, \theta^*)\, d\phi = 0$. This leads us to Proposition 1.

Proposition 1 (Run Cutoff). *In the limit $\sigma \to 0$, the unique equilibrium in $t = 1$ has unsecured lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = (1 - s)\, e^{-W\left(\frac{d-1}{\frac{k-s}{1-s}\left(1 - \frac{\ell}{1-k}\right)}\right)} + c - (k - s), \tag{4}$$

where all unsecured lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[13]

Proposition 1 allows us to derive the relation between the probability of bankruptcy and secured credit:

Corollary 1 (Rollover Yield, Repo, and Stability). *The run threshold θ^* has the following properties:*

(i) *it is strictly convex and strictly decreasing in d;*

(ii) *it is strictly concave in s;*

(iii) *it is first strictly increasing then strictly decreasing in s if the value of safe collateral is high ($k \geq 1/2$). If the value of safe collateral is low ($k < 1/2$), it is first strictly increasing then strictly decreasing in s if the rollover yield is high ($d > \underline{d} > 1$), and strictly decreasing if the rollover yield is low ($d \leq \underline{d}$).*

Figure 2: Yield and Repo Effects on Stability

(a) High Collateral $\left(k \geq \frac{1}{2}\right)$ (b) Low Collateral $\left(k < \frac{1}{2}\right)$
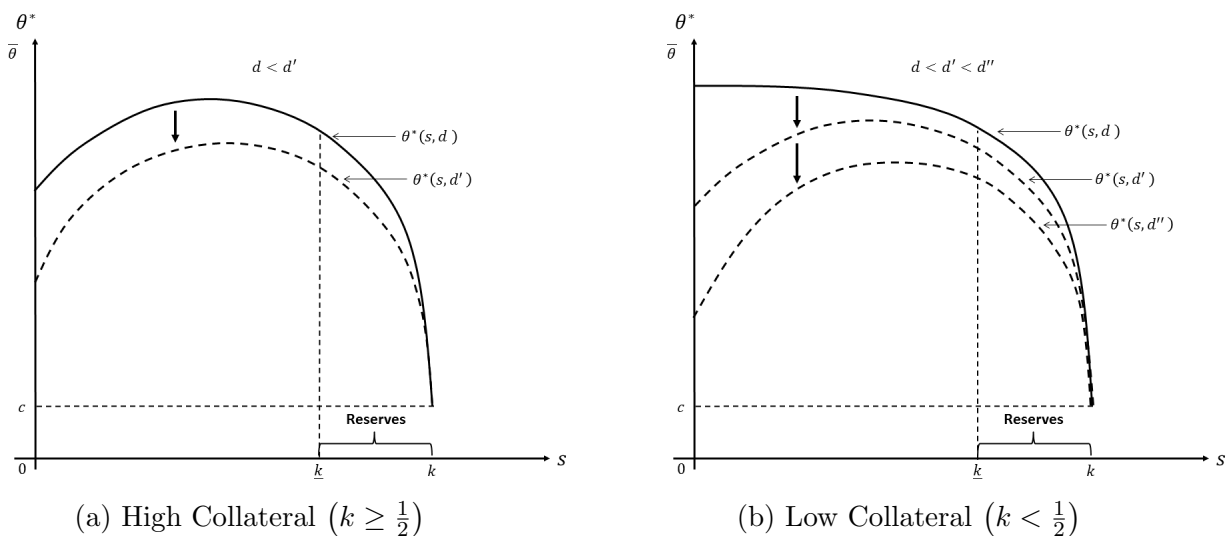
These results can be more easily interpreted after rewriting (4):

$$
\underbrace{\frac{\theta^* - c + (k - s)}{1 - s}}_{\text{unsecured debt recovery ratio}} = e^{-W\left(\overbrace{\frac{d - 1}{\frac{k-s}{1-s}\left(1 - \frac{\ell}{1-k}\right)}}^{\text{rolling over net-benefit-to-cost ratio}}\right)}. \tag{5}
$$

The effect of a higher rollover yield on unsecured debt d is quite intuitive, as it directly increases the reward to roll over and thus unambiguously discourages runs.

More repo credit has two effects: an increase in risk for unsecured debt in default and a reduction in liquid assets available to running lenders. More repo credit reduces asset liquidity and raises the chance that the bank runs out of reserves, thus increasing the appeal of running. This is equivalent to a deterioration in the quality of assets backing unsecured debt repayment. In addition the mandatory stay provision introduces a relative payoff effect. As runners can only be paid out of liquid assets, the relative run payoff drops when these become scarce, encouraging lenders to roll over. For most parameter values this produces a hump-shaped run threshold θ^* in s.[14]

---

[13]$W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

[14]This is quite different from the monotonic effect of fundamental risk on run frequency. See Matta and Perotti (2016) for a comparison with the classic result of Goldstein and Pauzner (2005).

The Corollary indicates that whenever liquid asset are abundant (k is large, so that $k \geq \frac{1}{2}$) a small increase in secured debt above $s = 0$ always leads to a higher risk of runs. This is the specular effect of increasing liquidity k available to runners. In this case the threshold θ^* will be at first rising and then falling in the amount of secured debt.

Figure 2(a) shows such a case. The probability of runs at first rises in s, reflecting the dominance of the risk concentration effect when available collateral is declining. It is then decreasing when the payoff effect becomes more prominent, as very little collateral is available to runners.

The run threshold $\theta^*(s, d)$ may also be downward sloping from $s = 0$ when k is sufficiently low and the rollover reward offered to unsecured debt d is very low. In Figure 2(b) as k is reduced the curve θ^* shifts leftwards relative to the y axis, so the effect of a lower k is that the chance of runs may be declining throughout in s. In the specific case of very high asset illiquidity it may be best to maximize the use of repo as it discourages withdrawals by subtracting liquid collateral from runners.

This last result should be interpreted with caution as it is implausible that intermediaries with only illiquid assets would be able to be funded with demandable debt. Overall the funding choice that reduces run frequency most directly is the choice of a high rollover yield d.

3.1 Funding

This section examines the bank's initial funding choice (s, d). To focus on runs driven by asset liquidity risk, we henceforth take $c \to 0$. Because the project has positive NPV for any funding choice, we can focus on the cost versus stability tradeoff.

The expected payoff of unsecured lenders as a function of its face value d is

$$V_U(s, d) = \lambda d + (1 - \lambda) \left[\left(\frac{\bar{\theta} - \theta^*(s, d)}{\bar{\theta}} \right) d + \frac{\theta^*(s, d)}{\bar{\theta}} \frac{k - s + \ell}{1 - s} \right] \tag{6}$$

The bank's payoff can be written as the value of the assets of a solvent bank r net of financing

costs, minus the expected deadweight loss $DW(s,d)$:

$$V_B(s,d) = \lambda \left[r - d(1-s) - s \right] + (1 - \lambda) \left(\frac{\overline{\theta} - \theta^*(s,d)}{\overline{\theta}} \right) \left[r - d(1-s) - s \right] \qquad (7)$$

$$= r - s - (1-s) V_U(s,d) - DW(s,d),$$

where $DW(s,d)$ is the total payoff lost in the event of bankruptcy, that is

$$DW(s,d) = (1 - \lambda) \frac{\theta^*(s,d)}{\overline{\theta}} (r - k - \ell). \qquad (8)$$

3.1.1 Socially Optimal Funding

The socially optimal financing contract chooses a pair (s,d) that maximizes the aggregate payoff subject to the participation constraint of the bank and unsecured lenders:

$$\max_{s,\, d} r - DW(s,d) \qquad (9)$$

subject to

$$V_B(s,d) \geq 0, \; V_U(s,d) \geq \gamma, \; s \in [0, \underline{k}].$$

This is equivalent to minimize the probability of bankruptcy $\theta^*(s,d)$ — and thus the deadweight loss of runs — subject to agents' participation constraints. Since the chance of runs $\theta^*(s,d)$ is decreasing in d, the social planner chooses to increase the rollover premium d as much as possible for any s. As the bank's participation constraint is binding at $d = \frac{r-s}{1-s}$ and its payoff is concave and decreasing, the value $d = \frac{r-s}{1-s}$ is the maximum face value feasible.

Thus under the socially optimal funding choice unsecured lenders receive positive rollover rents (i.e. above their participation constraint). The social optimum is achieved by minimizing $\theta \left(s, \frac{r-s}{1-s} \right)$ subject to $s \in [0, \underline{k}]$. The social planner can choose to increase the rollover reward $d = \frac{r-s}{1-s}$ by increasing the amount of secured debt s as this relaxes the bank's participation constraint. However this may come at the cost of directly increasing the probability of

13

Figure 3: Socially Optimal Funding

(a) High Asset Value $(r > \underline{r})$　　　　　(b) Low Asset Value $(r \leq \underline{r})$
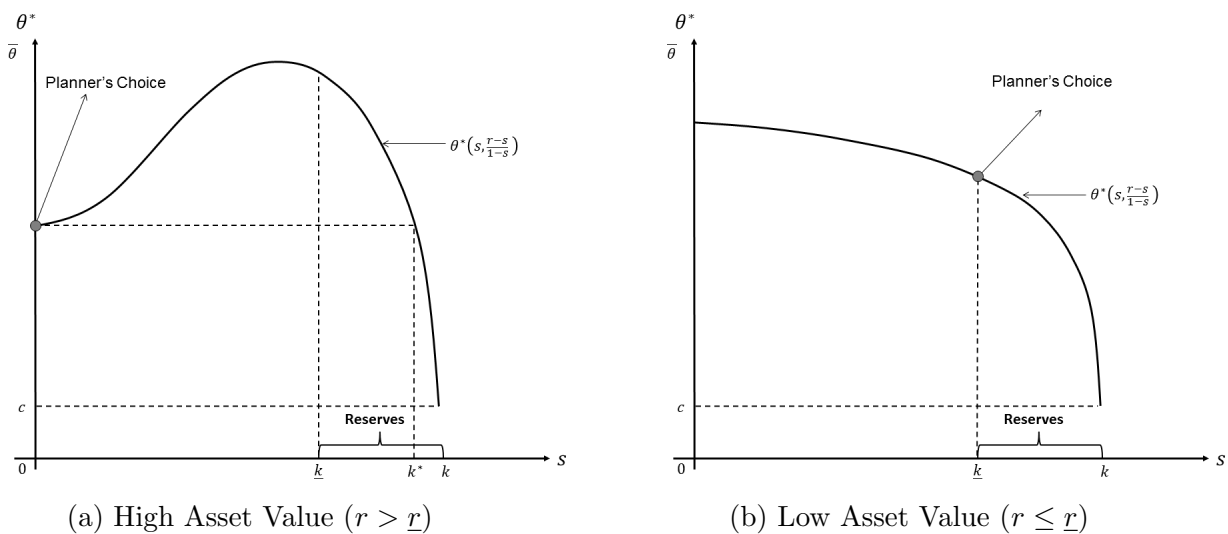
bankruptcy.

Proposition 2 below shows that the social planner either issues no secured debt or the maximum amount possible, a result that comes from the strict quasiconcavity of $\theta\left(s, \frac{r-s}{1-s}\right)$.

Proposition 2 (Optimal Funding). *The socially optimal contract denoted by (s^o, d^o) sets the rollover yield d^o such that the bank breaks even ($d^o = \frac{r-s^o}{1-s^o}$). The bank issues either only unsecured debt $(s^o, d^o) = (0, r)$ or the maximum possible amount of secured debt $(s^o, d^o) = \left(\underline{k}, \frac{r-\underline{k}}{1-\underline{k}}\right)$. If the value of the assets is high $(r > \underline{r} > 1)$ issuing secured debt is socially optimal if and only if the reserve requirement is sufficiently low $(\underline{k} > \underline{k}^* \in (0, k))$. If the asset value is low $(r \leq \underline{r})$ it is optimum to issue the maximum amount of secured debt to maximize the rollover premium subsidy.*

The slope of the threshold curve $\theta\left(s, \frac{r-s}{1-s}\right)$ at $s = 0$ depends on the return on assets. When r is high $(r > \underline{r} > 1)$ the social planner choice of a very high rollover premium d is very effective at reducing runs. We can show here that in this case (illustrated in Figure 3(a)) the threshold curve is first strictly increasing then decreasing. As a result, the social planner chooses to issue no secured debt. An exception is when the reserve requirement is so low that retained liquidity can be reduced to the point where runs are strongly discouraged.[15] When instead the

---

[15] A reserve requirement at 0 would be impossible in a more general version of this model where demandable

asset value is low $(r \leq \underline{r})$ the social planner choice of the rollover reward results in a downward sloping threshold $\theta \left(s, \frac{r-s}{1-s} \right)$ from $s = 0$. In this case it is best to maximize the use of repo, as it discourages withdrawals by subtracting liquid collateral from runners (see Figure 3(b)).

Overall, the result suggests that maximizing the rollover reward d while choosing $s = 0$ is most effective in reducing runs whenever the value of the assets enables to support a large rollover rent. Otherwise, the social planner will combine a maximum d with maximum repo funding, both to fund a higher d and to reduce liquidity for runners.

It is worth noting that the result of Proposition 2 does not imply that secured debt could not add value if $\underline{k} \leq \underline{k}^*$. If the project had positive NPV if and only if some secured debt is used $(r - \gamma < 0)$, then it could be financed only if some secured debt is used. Specifically, if

$$\lambda r + (1 - \lambda)(k + \ell) - (1 - \underline{k})\gamma - \underline{k} > 0,$$

then the project can only be financed if the bank issues enough secured debt.

3.1.2 Private Funding Choice

The bank's problem is to choose a funding structure (s, d) that maximize its payoff subject to the participation constraint:

$$\max_{s,\, d} V_B(s, d) \tag{10}$$

subject to

$$V_U(s, d) \geq \gamma, \; s \in [0, \underline{k}].$$

In choosing its optimal funding structure the bank faces a tradeoff between the cost of financing and the expected deadweight loss. The cost of financing is decreasing in the face value of unsecured debt d. As the unsecured lenders' required payoff is greater than for secured lenders, increasing the proportion of secured debt reduces the average cost of financing.

---

debt is an optimal contract to ensure withdrawals at $t = 1$ (Matta and Perotti (2016)).

However, lower d makes runs more likely, which increases the deadweight loss.

Proposition 3 (Private Inefficiency). *Let (s^*, d^*) be the solution to (10). The probability of bankruptcy under the socially optimal funding structure is lower than that under the bank's financing policy: $\theta^* (s^o, d^o) < \theta^* (s^*, d^*)$.*

Because the private choice of d is lower than for the social planner, in graphic terms it produces an upward shift of the $\theta^* (s^*, d)$ curve and a higher intercept at $s = 0$. The run threshold curve also exhibit increasing concavity. In conclusions, the private choice of s^* is either equal or higher than the social optimum value. Even when the private choice on the amount of repo debt is equal it is associated with a lower rollover yield d. Intuitively, shareholders prefer to earn more in solvent states than to minimize the chance of runs. This leads to a higher threshold $\theta^* (s^*, d)$ and thus more frequent runs than the social optimum.

Proposition 4 characterizes the optimal private funding choice. The interesting results arise when the chance of illiquidity is not too large (when λ is low), a realistic case.

Proposition 4 (Private Funding). *The bank's optimal funding policy is as follows:*

(i) *there exists a cutoff $\lambda_1 \in [0, 1)$ such that, if $\lambda \geq \lambda_1$ the bank's financing policy (s^*, d^*) has the bank borrowing either by issuing only unsecured debt $(s^* = 0)$ or by issuing the maximum possible amount of secured debt $(s^* = \underline{k})$. In this case the optimal face value of unsecured debt d^* is characterized by either $\mu^* [V_U (s^*, d^*) - \gamma] = 0$ or $-\frac{\partial DW(s^*, d^*)}{\partial d} = \frac{\partial V_U(s^*, d^*)}{\partial d} [1 - s^* - \mu^*]$, where μ^* is the Lagrange multiplier associated with unsecured lenders' participation constraint;*

(ii) *There exists a cutoff $\lambda_2 \in (0, 1)$ such that if $\lambda > \lambda_2$ the rollover yield is set equal to unsecured lenders' participation constraint $(V_U (s^*, d^*) - \gamma = 0)$;*

(iii) *There exists a cutoff $\lambda_3 \in [0, 1)$ such that, if $\lambda > \max \{\lambda_1, \lambda_2, \lambda_3\}$, the bank borrows by issuing the maximum possible amount of secured debt $(s^* = \underline{k})$. The chosen rollover yield is set equal to unsecured lenders' participation constraint $(V_U (s^*, d^*) - \gamma = 0)$.*

16

The face value of unsecured debt balances the lower cost of funding against the higher expected deadweight loss from reducing d subject to the participation constraint. The optimal funding structure when λ is sufficiently high is a corner solution because, in this case, the bank's payoff is quasiconvex in s.

4 Deposit Insurance

This section discusses the possibility of third party deposit insurance (DI) for unsecured lenders. Consistent with real practice we model DI as a minimum payment of $\pi \in [0,1]$ for unsecured lenders in all states.

In the presence of DI the payoff of unsecured lenders who do not roll over in $t = 1$ is

$$\pi_U^N(\phi, \theta) = \begin{cases} 1, & \text{if no bankruptcy} \\ \frac{1-\phi^*}{1-\phi} + \left(1 - \frac{1-\phi^*}{1-\phi}\right) \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if bankruptcy} \end{cases},$$

while that of those who roll over is

$$\pi_U^R(\phi, \theta) = \begin{cases} qd + (1-q) \max\left\{\frac{\rho - s - (1-s)(1-\phi)}{\phi(1-s)}, \pi\right\}, & \text{if no bankruptcy} \\ \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & \text{if bankruptcy} \end{cases}.$$

Therefore, unsecured lender's net payoff of rolling over relative to that of running is

$$\Pi_U^R(\phi, \theta) = \begin{cases} qd + (1-q) \max\left\{\frac{\rho - s - (1-s)(1-\phi)}{\phi(1-s)}, \pi\right\} - 1, & \text{if no bankruptcy} \\ -\frac{k-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}\right), & \text{if bankruptcy} \end{cases}. \tag{11}$$

As in Diamond and Dybvig (1983) if the regulator provides full insurance ($\pi = 1$) then it is a dominant strategy to roll over regardless of the uncertain liquidation value of the assets θ and the fraction of unsecured lenders that roll over ϕ. Thus full insurance deters runs and achieves efficiency (since all runs are inefficient in the limit as c goes to zero). If the amount

17

of DI is such that $\pi \leq \min \left\{\frac{\ell}{1-k}, \rho\right\}$ the payoffs are the same as those without the presence of DI and all the previous results go through.

We are thus left with the following two cases: $\min \left\{\frac{\ell}{1-k}, \rho\right\} < \pi \leq \max \left\{\frac{\ell}{1-k}, \rho\right\}$ and $\max \left\{\frac{\ell}{1-k}, \rho\right\} < \pi < 1$. As before, the equilibrium cutoff θ^*_{DI} can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ: $\int_0^1 \Pi^R_U (\phi, \theta^*_{DI}) = 0$.

This leads us to Proposition 5:

Proposition 5 (Run Cutoff with DI). *Suppose* $\min \left\{\frac{\ell}{1-k}, \rho\right\} < \pi < 1$. *In the limit* $\sigma \to 0$ *the unique equilibrium in* $t = 1$ *has unsecured lenders following monotone strategies with threshold* θ^* *given by*

$$\theta^*_{DI} = (1-s) e^{-W\left(\frac{d-1}{\frac{k-s}{1-s}\left(1-\max\left\{\frac{\ell}{1-k}, \pi\right\}\right)}\right)} + c - (k-s), \tag{12}$$

where all unsecured lenders roll over if $\theta > \theta^*$ *and do not roll over if* $\theta < \theta^*$.

The results in Corollary 2 below follow from Proposition 5.

Corollary 2 (DI Effect on Stability). *If* $\pi = 1$, *then there is no run in the presence of DI. If* $\pi \leq \min \left\{\frac{\ell}{1-k}, \rho\right\}$, *the probability of bankruptcy with and without DI are the same:* $\theta^*_{DI} = \theta^*$. *If* $\min \left\{\frac{\ell}{1-k}, \rho\right\} < \pi < 1$, *the probability of bankruptcy with DI is at least as low as that without DI:* $\theta^*_{DI} = \theta^*$ *for* $\pi \leq \frac{\ell}{1-k}$ *and* $\theta^*_{DI} < \theta^*$ *for* $\pi > \frac{\ell}{1-k}$, *in which case* θ^*_{DI} *is strictly decreasing in* π.

The results above show that for any given private funding choice, an increase in the level of DI from π to $\pi' > \pi$ reduces the probability of bankruptcy (provided that π is sufficiently large). The natural question that arises is whether the same result holds when the bank takes into account the effect of DI on run frequency.

If the high state is sufficiently likely (λ large enough), then Proposition 4 tells us that the bank issues the maximum possible amount of secured debt $s^* = \underline{k}$ and minimizes the rollover premium on unsecured debt to the unsecured lenders' participation constraint $V_U (\underline{k}, d^*; \pi) = \gamma$. An increase in π *directly reduces* the probability of bankruptcy as $\theta^* (\underline{k}, d^*; \pi') < \theta^* (\underline{k}, d^*; \pi)$,

18

which increases unsecured lenders' expected payoff $V_U\left(\underline{k}, d^*; \pi'\right) > V_U\left(\underline{k}, d^*; \pi\right) = \gamma$. Thus the bank's is able to reduce the face value of debt to $d^{*\prime} < d^*$ such that $V_U\left(\underline{k}, d^{*\prime}; \pi'\right) = V_U\left(\underline{k}, d^*; \pi\right) = \gamma$, which *indirectly increases* the probability of bankruptcy: $\theta^*\left(\underline{k}, d^{*\prime}; \pi\right) > \theta^*\left(\underline{k}, d^*; \pi\right)$. Corollary 3 below shows that the direct effect dominates when λ is large enough.

Corollary 3 (DI Effect on Private Inefficiency). *Suppose* $\frac{\ell}{1-k} < \pi < 1$ *and* λ *is sufficiently large. Then under the private funding choice with DI both the face value of unsecured debt* d^* *and the probability of bankruptcy* $\theta^*\left(\underline{k}, d^*\right)$ *are strictly decreasing in* π.

The intuition behind the result in Corollary 3 is simple. If the high state is sufficiently likely, then unsecured lenders' ex ante payoff is highly sensitive to the rollover premium. Therefore a small drop in the face value d rapidly offsets the gains brought about by decreases in probability of bankruptcy. As a result, the bank is unable to significantly reduce the face value of unsecured debt following an increase in the level of DI.

5 Conclusion

This paper has examined the effect of secured financial credit on run incentives. Secured debt can be designed so as it is always rolled over, as the pledge of liquid assets ensures its safety even in default. In the absence of strategic complementarity, insuring risk intolerant lenders is efficient and reduces funding costs, while unsecured debt is compensated by a higher yield.

Our main result is that repo debt affects coordination among other lenders' choices, by altering the allocation of risk as well as liquidity. The ultimate effect may be a reduction or an increase in run frequency. Our analysis highlights the trade off. As secured credit receives a safe and liquid pledge, it shifts some risk to unsecured debt, which will require a higher rollover yield. Subtracting safer assets from the resources available to unsecured debt is equivalent to an increase in fundamental risk and thus leads to more frequent runs (Goldstein and Pauzner (2005)). However, pledging liquid assets also reduces the relative return to running creditors, because of the effect of mandatory stay (Matta and Perotti (2016)). The net effect on run

incentives is thus ambiguous. In addition, the lower cost of repo funding could be used in principle to subsidize a larger rollover premium, thus reducing run incentives.

We show that in the unique run equilibrium the choice of secured debt by a profit maximizing bank tends to result in more frequent unsecured runs than in the social optimum. Intuitively, as long as illiquidity is sufficiently rare, intermediaries seek inexpensive repo funding even if they internalize the increased risk of inefficient runs of unsecured debt.

In conclusion, the use of secured funding has a beneficial effect to provide absolute safety, in the aggregate mobilizing more savings and reducing the cost of credit. At the same time, its unconstrained private use may be excessive relative to the social optimum because of its effect on the stability of other funding sources. The overall effect may be procyclical on both credit volume as well as risk incentives. This makes a clear case for greater regulatory awareness about its use.

The direct effects of repo debt we describe complements the known indirect effect from collateral fire sales (Duffie and Skeel (2012)). As these sales reduce collateral value and its liquidity, risk intolerant repo lenders would require higher haircuts. We are able to show that higher margins leads to higher collateralization and a further boost to the frequency of runs.

A final question for future research concerns the effect of encumbered assets on stability when disclosure is limited about secured transactions, such as repo debt and derivatives (Acharya and Bisin (2014)). Imprecise disclosure may create Knightian uncertainty and self fulfilling panics (Caballero and Khrisnamurthy (2008)), and may induce unsecured debt to become information sensitive and thus insecure (run-prone), as in Gorton and Ordoñez (2014).

Appendix

Proof of Proposition 1. Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games, including those where θ is drawn from a uniform distribution on $[\underline{\theta}, \overline{\theta}]$, the noise terms η_i are i.i.d. across players and drawn from a uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$, and that satisfy the following additional conditions: (i) for each θ, there exists $\phi^* \in \mathbb{R} \cup \{-\infty, \infty\}$ such that $\Pi_U^R(\phi, \theta) > 0$ if $\phi > \phi^*$ and $\Pi_U^R(\phi, \theta) < 0$ if $\phi < \phi^*$; (ii) $\Pi_U^R(\phi, \theta)$ is nondecreasing in θ; (iii) there exists a unique θ^* that satisfies $\int_0^1 \Pi_U^R(\phi, \theta^*) d\phi = 0$; (iv) there exists $\overline{D}$ and $\underline{D}$ with $\sigma < \min\{\overline{\theta} - \overline{D}, \underline{D} - \underline{\theta}\}$, and $\epsilon > 0$ such that $\Pi_U^R(\phi, \theta) \leq -\epsilon$ for all $\phi \in [0, 1]$ and $\theta \leq \underline{D}$ and $\Pi_U^R(\phi, \theta) > \epsilon$ for all $\phi \in [0, 1]$ and $\theta \geq \overline{D}$; and (v) continuity of $\int_0^1 w(\phi) \Pi_U^R(\phi, x) d\phi$ with respect to signal x and density w. Except for (iii), $\Pi_U^R(\phi, \theta)$ clearly satisfies (i), (ii), (iv) and (v).

Let $\Delta(\theta; s, d) \equiv \int_0^1 \Pi_U^R(\phi, \theta) d\phi$. Since $\Delta(\theta; s, d) < 0$ for all (s, d) and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Moreover, since $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$, we must show that $\Delta(c; s, d) \leq 0$ for all (s, d) (otherwise for some (s, d) we have $\Delta(\theta; s, d) \geq \Delta(c; s, d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*; s, d) = 0$). It is straightforward to show that (a) $\Delta(c; s, d)$ is strictly increasing in d, (b) d is bounded by $\frac{r-s}{1-s}$ (in which case the bank's participation constraint binds), (c) $\Delta\left(c; s, \frac{r-s}{1-s}\right)$ is decreasing in s if $\frac{r-1}{1-\ell-k} \leq \frac{1}{k}$, and (d) that $\Delta(c; 0, r) = k\left(1 - \frac{\ell}{1-k}\right) \ln \frac{k}{e^{-\frac{r-1}{1-\frac{\ell}{1-k}}}} \leq (<)\, 0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-k}}} \geq (>)\, k$. Therefore, for $\frac{r-1}{1-\ell-k} \leq 1$ we have

$$
e^{-\frac{r-1}{1-\frac{\ell}{1-k}}} > 1 - \frac{r-1}{1-\frac{\ell}{1-k}} = (1-k)\left(1 - \frac{r-1}{1-\ell-k}\right) + k \geq k,
$$

which implies that for all (s, d) we have $\Delta(c; s, d) \leq \Delta\left(c; s, \frac{r-s}{1-s}\right) \leq \Delta(c; 0, r) < 0$. In addition, for all (s, d) we have $\Delta(\theta; s, d) > 0$ for θ sufficiently large such that there exists $\theta^* \geq c$ that satisfies $\Delta(\theta^*; s, d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$.

The cutoff θ^* can be derived by solving $\int_0^1 \Pi_U^R(\phi, \theta^*) d\phi = 0$, which is equivalent to:

$$
\frac{k-s}{1-s}\left(1 - \frac{\ell}{1-k}\right) \ln \frac{\theta^* - c + k - s}{1-s} + \frac{\theta^* - c + k - s}{1-s}(d-1) = 0. \tag{A.1}
$$

After some algebra, (A.1) can be rewritten as

$$
\frac{d-1}{\frac{k-s}{1-s}\left(1 - \frac{\ell}{1-k}\right)} = -\ln \frac{\theta^* - c + k - s}{1-s} e^{-\ln \frac{\theta^* - c + k - s}{1-s}}. \tag{A.2}
$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (the Lambert W function), that is, $x = W(y)$. Combined with (A.2) this implies $\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{k-s}{1-s}\left(1-\frac{\ell}{1-k}\right)}\right)} + c - (k-s)$. $\quad\square$

Proof of Corollary 1. First, let us establish a couple of results relative to the W function. Implicitly differentiating $y = W(y)\, e^{W(y)}$ results in

$$W' = \frac{W}{(W+1)\, y} = \frac{e^{-W}}{1+W} > 0,$$
$$W'' = W'^2 \left(\frac{-2-W}{1+W}\right) < 0.$$

Differentiating θ^* with respect to d and s shows (i) and (ii):

$$\frac{\partial \theta^*}{\partial d} = \frac{-(1-s)\, e^{-W} W'}{\frac{k-s}{1-s}\left(1-\frac{\ell}{1-k}\right)} < 0, \tag{A.3}$$

$$\frac{\partial^2 \theta^*}{\partial d^2} = \frac{(1-s)\, e^{-W}\left(W'^2 - W''\right)}{\left[\frac{k-s}{1-s}\left(1-\frac{\ell}{1-k}\right)\right]^2} > 0, \tag{A.4}$$

$$\frac{\partial \theta^*}{\partial s} = e^{-W}\left[e^W - 1 - \frac{1-k}{k-s}\frac{W}{W+1}\right], \tag{A.5}$$

$$\frac{\partial^2 \theta^*}{\partial s^2} = -\frac{e^{-W} W(-2-W)}{(W+1)^3}\frac{(1-k)^2}{(k-s)^2(1-s)} < 0. \tag{A.6}$$

We now show (iii). Since $\lim_{s \to k}\frac{\partial \theta^*(s,d)}{\partial s} = -\infty$ and $\theta^*(s,d)$ is strictly concave in s, it follows that $\theta^*(s,d)$ is strictly decreasing in s if $\frac{\partial \theta^*(0,d)}{\partial s} \leq 0$, and first strictly increasing then decreasing in s if $\frac{\partial \theta^*(0,d)}{\partial s} > 0$. Let us write $\frac{\partial \theta^*(0,d)}{\partial s} = e^{-W(d)}\beta(d)$, where $\beta(d)$ is the term inside the brackets in (A.5).

We have that $\beta'(d) = W'(d)\left[e^{W(d)} - \frac{1-k}{k}\frac{1}{(W(d)+1)^2}\right] > 0$ whenever $k \geq \frac{1}{2}$. Since $\beta(1) = 0$, it follows that $\beta(d) > 0 \Rightarrow \frac{\partial \theta^*(0,d)}{\partial s} > 0$ for all $d > \underline{d} = 1$ and $k \geq \frac{1}{2}$, which implies $\theta^*(s,d)$ is first strictly increasing then decreasing.

For $k < \frac{1}{2}$, we have that $\beta'(d) < 0$ for d close enough to 1 and $\beta'(d) > 0$ for d high enough, which implies there exists $d' > 1$ such that $\beta'(d') = 0$. Our next step is to show that $\beta(d)$ is strictly quasi-convex. We do so by showing that the strict single crossing functions $W'(d)\, e^{W(d)}$ and $-W'(d)\frac{1-k}{k}\frac{1}{(W(d)+1)^2}$ satisfy strict signed-ratio monotonicity, which implies $\beta'(d)$ is a strict single crossing function (Quah and Strulovici, 2012). Two functions $f(d)$ and $g(d)$ satisfy strict signed-ratio monotonicity if whenever $f(d) > 0$ and $g(d) < 0$, $-\frac{g(d)}{f(d)}$ is strictly decreasing and whenever $f(d) < 0$ and $g(s) > 0$, $-\frac{f(d)}{g(d)}$ is strictly decreasing. We take $g(d) = -W'(d)\frac{1-k}{k}\frac{1}{(W(d)+1)^2}$ and $f(d) = W'(d)\, e^{W(d)}$. Since $f(d)$ is always positive, we only

22

need to consider the case in which $g(d) < 0$. In this case, $-\frac{g(d)}{f(d)} = \frac{\frac{1-k}{k}\frac{1}{(W(d)+1)^2}}{e^{W(d)}}$ is clearly strictly decreasing since the numerator is strictly decreasing while the denominator is strictly increasing.

Therefore, $\beta'(d) > 0$ for $d > d'$ and $\beta'(d) < 0$ for $d < d'$. Since $\beta(1) = 0$, it follows that $\beta(d) < 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} < 0$ for $1 < d \leq d'$. Because $\beta(d) > 0$ for d sufficiently high, there exists $\underline{d} > d'$ such that $\beta(\underline{d}) = 0 \Rightarrow \frac{\partial\theta^*(0,\underline{d})}{\partial s} = 0$. As a result, we have $\beta(d) \leq 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} \leq 0$ for $d \leq \underline{d}$ ($\theta^*(s,d)$ is strictly decreasing) and $\beta(d) > 0 \Rightarrow \frac{\partial\theta^*(0,d)}{\partial s} > 0$ for $d > \underline{d}$ ($\theta^*(s,d)$ is first strictly increasing then decreasing). $\square$

Proof of Proposition 2. We first show that the bank's participation constraint must bind at a solution (s^o, d^o). Suppose not, that is, $V_B(s^o, d^o) > 0$. The aggregate payoff $r - DW(s,d)$ is increasing in d, while the bank's payoff is either one of the following: (1) decreasing, or (2) increasing and then decreasing. To see this, note that

$$\overline{\theta}\frac{\partial V_B(s,d)}{\partial d} = -(1-s)\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r - d(1-s) - s)\frac{\partial\theta^*}{\partial d} - (\overline{\theta} - 1)(1-s)$$

is negative for $d = \frac{r-s}{1-s}$ and $\overline{\theta}\frac{\partial^2 V_B(s,d)}{\partial d^2} < 0$. If $\frac{\partial V_B(s,d)}{\partial d} \leq 0$ for all d, then $V_B(s,d)$ is monotone decreasing. If $\frac{\partial V_B(s,d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(s,d)}{\partial d} = 0$. Since $V_B(s,d)$ is strictly concave, $\frac{\partial V_B(s,d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = \frac{r-s}{1-s}$. Therefore, the social planner can increase d^o until $V_B(s^o, d^o)$ binds: this increases the aggregate payoff while still satisfying the constraints, which contradicts (s^o, d^o) being a solution.

The result that the bank's participation constraint binds along with our assumption that the project has positive NPV implies that the unsecured lenders' participation constraint does not bind. Thus, the social social planner's problem can be equivalently rewritten as $\min_{s\in[0,p\underline{k}]}$ $\theta^*\left(s, \frac{r-s}{1-s}\right)$.

We now show that $\theta^*\left(s, \frac{r-s}{1-s}\right)$ is strictly quasi-concave in s, implying a corner solution $s^o \in \{0, p\underline{k}\}$. We do so by showing that the strict single crossing functions $e^{-W}\frac{1-k}{k-s}\frac{W}{W+1}$ and $-e^{-W}\left(e^W - 1 - \frac{W}{W+1}\right)$ satisfy strict signed-ratio monotonicity, which implies $-\frac{\partial\theta^*\left(s,\frac{r-s}{1-s}\right)}{\partial s} = e^{-W}\left[\frac{1-k}{k-s}\frac{W}{W+1} - \left(e^W - 1 - \frac{W}{W+1}\right)\right]$ is a strict single crossing function (Quah and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy strict signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is strictly decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is strictly decreasing. We take $g(s) = -e^{-W}\left(e^W - 1 - \frac{W}{W+1}\right)$ and $f(s) = e^{-W}\frac{1-k}{k-s}\frac{W}{W+1}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it

must be that $-\frac{g(s)'f(s)-g(s)f(s)'}{f(s)^2} < 0$. Indeed, we have

$$
\begin{aligned}
-g\left(s\right)'f\left(s\right)+g\left(s\right)f\left(s\right)' &= \frac{\left(1-k\right)}{\left(k-s\right)^2}\left[1-\frac{\left(e^W-1\right)\left(W+2\right)}{W\left(W+1\right)}\right] \\
&< \frac{\left(1-k\right)}{\left(k-s\right)^2}\left[1-\frac{W+2}{W+1}\right] < 0,
\end{aligned}
$$

where the first inequality results from the following version of Bernoulli's inequality: $e^x > 1+x$ for $x > 0$.

Finally, we determine the conditions under which each corner solution is optimal. Since $\theta^*\left(s,\frac{r-s}{1-s}\right)$ is strictly quasi-concave and $\lim_{s\to k}\frac{\partial\theta^*\left(s,\frac{r-s}{1-s}\right)}{\partial s} = -\infty$ it follows that θ^* is strictly decreasing in s if $\frac{\partial\theta^*(0,r)}{\partial s} \le 0$, and first strictly increasing then decreasing in s if $\frac{\partial\theta^*(0,r)}{\partial s} > 0$, where

$$
\frac{\partial\theta^*\left(0,r\right)}{\partial s} = e^{-W}\left[e^W-1-\frac{1}{k}\frac{W}{W+1}\right]. \tag{A.7}
$$

Let us write $\frac{\partial\theta^*(0,r)}{\partial s} = e^{-W(r)}\beta\left(r\right)$, where $\beta\left(r\right)$ is the term inside the brackets in (A.7). Following the same steps in the proof of Corollary 3 (for the case when $k < \frac{1}{2}$), we can show that $\beta\left(r\right)$ is strictly quasi-convex and there exists $\underline{r} > 1$ is such that $\beta\left(\underline{r}\right) = 0$, with $\beta\left(r\right) \le 0 \Rightarrow \frac{\partial\theta^*(0,r)}{\partial s} \le 0$ for $r \le \underline{r}$ and $\beta\left(r\right) > 0 \Rightarrow \frac{\partial\theta^*(0,r)}{\partial s} > 0$ for $r > \underline{r}$.

Therefore, it follows that $(s^o,d^o) = \left(\underline{k},\frac{r-\underline{k}}{1-\underline{k}}\right)$ for $r \le \underline{r}$. For $r > \underline{r}$, there exists s' such that $\theta^*\left(s,\frac{r-s}{1-s}\right)$ is strictly increasing for $s < s'$ and strictly decreasing for $s > s'$. From the proof of Proposition 1, we know that $\theta^*\left(0,r\right) > c$. Since $\lim_{\underline{k}\to k}\theta^*\left(\underline{k},\frac{r-\underline{k}}{1-\underline{k}}\right) = c$, there exists a $\underline{k}^* \in (s',k)$ such that $\theta^*\left(0,r\right) = \theta^*\left(\underline{k}^*,\frac{r-\underline{k}^*}{1-\underline{k}^*}\right)$, with $\theta^*\left(0,r\right) \le \theta^*\left(\underline{k},\frac{r-\underline{k}}{1-\underline{k}}\right)$ for $\underline{k} \le \underline{k}^*$ and $\theta^*\left(0,r\right) > \theta^*\left(\underline{k},\frac{r-\underline{k}}{1-\underline{k}}\right)$ for $\underline{k} > \underline{k}^*$. $\qquad\square$

Proof of Proposition 3. Suppose that $\theta^o\left(s^o,d^o\right) \ge \theta^*\left(s^*,d^*\right)$. Since we assume the project has positive NPV, the bank's payoff under (10) is greater than zero (the bank can guarantee a positive payoff by choosing $s = 0$). But then a contract (s^*,d) with d marginally greater than d^* satifies both participation constraints in (9) and results in $\theta^o\left(s^o,d^o\right) \ge \theta^*\left(s^*,d^*\right) > \theta^*\left(s^*,d\right)$. But this contradicts (s^o,d^o) being a solution to (9). $\qquad\square$

Proof of Proposition 4. We first show (i). We use the Principle of Iterated Suprema to break the bank's problem into two stages, that is, we solve $\max_{d\in D}\left[\max_{s\in S} V_B\left(s,d\right)\right]$, where $S = [0,\underline{k}]$ and $D = \{d : V_U\left(s^*\left(d\right),d\right) \ge \gamma\}$.

The next step is to show that $V_B\left(s,d\right)$ is quasiconvex in s if λ sufficiently high, which implies that there is not interior solution to problem $\max_{s\in S} V_B\left(s,d\right)$. This is done by showing

24

that $\frac{\partial V_B(s,d)}{\partial s} = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s} - \frac{\partial DW(s,d)}{\partial s}$ is a single crossing function, which is equivalent to $V_U - 1 - (1-s)\frac{\partial V_U}{\partial s}$ and $-\frac{\partial DW}{\partial s}$ satisfying signed-ratio monotonicity (Qua and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is decreasing. We take $g(s) = -\frac{\partial DW(s,d)}{\partial s}$ and $f(s) = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it must be that $-\frac{g(s)'f(s)-g(s)f(s)'}{f(s)^2} < 0$. We have

$$\bar{\theta}\left[-g(s)'f(s) + g(s)f(s)'\right] =$$

$$(1-\lambda)\theta^{*\prime\prime}(r-k-\ell)\left\{(d-1)\left[\bar{\theta} - (1-\lambda)\theta^*\right] + (1-\lambda)\theta^{*\prime}\left[d(1-s) - (k-s+\ell)\right]\right\}$$

$$-(1-\lambda)\theta^{*\prime}(r-k-\ell)\left\{(1-\lambda)\theta^{*\prime\prime}\left[d(1-s) - (k-s+\ell)\right] - 2(1-\lambda)\theta^{*\prime}(d-1)\right\}$$

$$= (1-\lambda)(r-k-\ell)(d-1)\left[\theta^{*\prime\prime}\left(\bar{\theta} - (1-\lambda)\theta^*\right) + 2(1-\lambda)\theta^{*\prime}\theta^{*\prime}\right].$$

The sign of the above expression is determined by the term inside brackets, which is strictly decreasing in λ. For any given s, it is negative if λ is sufficiently close to 1, so that we are left with two possibilities: either it is nonpositive for all λ, or there exists $\lambda(s) \in (0,1)$ such that it is nonpositive if $\lambda \geq \lambda(s)$ and positive if otherwise. If the former is true for all s, then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = 0$. Suppose there exists s such that the latter is true and denote X the set of all such s. Then $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1 = \sup\{\lambda(s) : s \in X\}$. Combining both cases we have that there exists a cutoff $\lambda_1 \in [0,1)$ such that $V_B(s,d)$ is quasiconvex if $\lambda \geq \lambda_1$, which in turn implies that we must have a corner solution: $s^* \in \{0, \underline{k}\}$.

We now turn to the problem $\max_{d \in D} V_B(s^*, d)$. The first order necessary conditions (FOC) are

$$-\frac{\partial DW(s^*, d)}{\partial d} = \frac{\partial V_U(s^*, d)}{\partial d}[1 - s^* - \mu], \tag{A.8}$$

$$\mu[V_U(s^*, d) - \gamma] = 0, \tag{A.9}$$

$$V_U(s^*, d) \geq \gamma, \tag{A.10}$$

$$\mu \geq 0. \tag{A.11}$$

To conclude the proof of (i) we need to show that any d satisfying the FOC is a global maximizer. This follows from

$$\bar{\theta}\frac{\partial V_B^2(s,d)}{\partial d^2} = 2(1-s)(1-\lambda)\frac{\partial \theta^*}{\partial d} - (1-\lambda)(r - d(1-s) - s)\frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which implies that $V_B(s^*, d)$ is (strictly) concave in d.

We now show (ii). Note that

$$\frac{\partial DW(s^*, d)}{\partial d} = \frac{(1-\lambda)}{\bar{\theta}} \frac{\partial \theta^*(s^*, d)}{\partial d}(r - k - \ell), \qquad (A.12)$$

$$\frac{\partial V_U(s^*, d)}{\partial d} = 1 - \frac{(1-\lambda)}{\bar{\theta}}\left[\theta^*(s^*, d) + \frac{\partial \theta^*(s^*, d)}{\partial d}\left(d - \frac{k - s^* + \ell}{1 - s^*}\right)\right]. \qquad (A.13)$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.8) approach 0 ((A.12) approximates 0) and $1 - s^*$ ((A.13) converges to 1), respectively. Since s is bounded above by $\underline{k} < 1$, the right-hand side of (A.8) is bounded away from 0. Therefore, there are only two possibilities: either the left-hand side of (A.8) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all λ, or there exists $\lambda(s^*, d) \in (0, 1)$ such that the left-hand side of (A.8) is smaller than the right-hand side if $\lambda > \lambda(s^*, d)$ and at least as great if otherwise. If the former is true for all d, then (A.8) can only be satisfied if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_2 = \sup\{\lambda(s^*, d) : d \in Y\}$, then (A.8) can only be satisfied if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_2 \in (0, 1)$ such $\mu > 0$ if $\lambda > \lambda_2$, which in turn implies that $V_U(s^*, d) - \gamma = 0$ (from (A.9)).

We finally show (iii). Suppose $\lambda > \max\{\lambda_1, \lambda_2\}$. In this case we know from (i) that there are two possible candidates for the bank's choice of secured debt: either $s^* = \underline{k}$ or $s^* = 0$. We also know that unsecured lenders' participation constraint binds. Therefore, the bank's implied payoffs are given by

$$V_B(\underline{k}, d^*(\underline{k})) = r - \underline{k} - (1 - \underline{k})\gamma - DW(\underline{k}, d^*(\underline{k})), \qquad (A.14)$$

$$V_B(0, d^*(0)) = r - \gamma - DW(0, d^*(0)). \qquad (A.15)$$

The difference is given by

$$V_B(\underline{k}, d^*(\underline{k})) - V_B(0, d^*(0)) = \underline{k}(\gamma - 1) - [DW(\underline{k}, d^*(\underline{k})) - DW(0, d^*(0))], \qquad (A.16)$$

which is positive for λ sufficiently close to 1. Thus, there are two cases to consider: either (A.16) (strictly increasing in λ) is nonnegative for all $\lambda \geq \lambda_3 = 0$, or there exists $\lambda_3 \in (0, 1)$ such that (A.16) is nonnegative if $\lambda > \lambda_3$, and negative if $\lambda < \lambda_3$. Therefore, we conclude that if $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, then the bank's financing policy has the bank borrowing by issuing only secured debt ($s^* = \underline{k}$) and d^* is such that $V_U(s^*, d) - \gamma = 0$. $\qquad \square$

Proof of Proposition 5. Proof is analogous to that of Propositions 1. $\qquad \square$

Proof of Corollary 2. See discussion in text. $\square$

Proof of Corollary 3. For λ large enough, unsecured lenders' participation constraint binds: $V_U\left(\underline{k}, d^*; \pi\right) = \gamma$. Thus, the overall change in θ^* caused by an increase in π can be found by differentiating both sides with respect to π, whihc yields:

$$\frac{\partial \theta^*\left(d, \pi\right)}{\partial d} d' + \frac{\partial \theta^*\left(d, \pi\right)}{\partial \pi} = \frac{\frac{(1-\lambda)\theta^*(d,\pi)}{\bar{\theta}}\left[d' - \frac{1-k}{1-\underline{k}}\right] - d'}{\frac{(1-\lambda)}{\bar{\theta}}\left[-\left(d-1\right) - \frac{1-k}{1-\underline{k}}\left(1-\pi\right)\right]}.$$

Since the denumerator on the right-hand side is negative and $d' < 0$, the overall expression is negative for λ large enough. $\square$

References

[1] Acharya, V. and A. Bisin, 2014, "Counterparty Risk Externality: Centralized Versus Over-the-counter Markets," *Journal of Economic Theory* 149, 153–182.

[2] Ahnert, T. and E. Perotti, 2016, "Cheap but Flightly: How Global Imbalances Affect Financial Stability," University of Amsterdam Working Paper.

[3] Ahnert, T., K. Anand, P. Gai, and J. Chapman, 2016, "Asset Encumbrance, Bank Funding and Financial Fragility," Bank of Canada Working Paper.

[4] Auh, J. and S. Sundaresan, 2015, "Repo Rollover Risk and the Bankruptcy Code," Columbia University Working Paper.

[5] Brunnermeier, M., 2009, "Deciphering the Liquidity and Credit Crunch 2007–2008," *Journal of Economic Perspectives* 23, 77–100.

[6] Brunnermeier, M. and M. Oehmke, 2013, "The Maturity Rat Race," *Journal of Finance* 68, 483–521.

[7] Caballero, R. J and E. Farhi, 2013, "A Model of the Safe Asset Mechanism (SAM): Safety Traps and Economic Policy," MIT Working Paper.

[8] Caballero, R. and A. Krishnamurthy, 2009, "Global Imbalances and Financial Fragility," *American Economic Review* 99, 584–588.

[9] Caballero, R. and A. Krishnamurthy, 2008,"Collective Risk Management in a Flight to Quality Episode," *Journal of Finance* 63, 2195–2230.

[10] Diamond, D. and P. Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Econommy* 91, 401–419.

[11] Gennaioli N, A. Shleifer and R. Vishny, 2013, "A Model of Shadow Banking," *Journal of Finance* 68, 1331–63.

[12] Goldstein, I. and A. Pauzner, 2005, "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance* 60, 1293–1328.

[13] Gorton, G. and A. Metrick, 2012, "Securitized Banking and the Run on Repo," *Journal of Financial Economics* 104, 425–51.

[14] Gorton, G. and G. Ordoñez, 2014, "Collateral Crises," *American Economic Review* 104, 343–78.

[15] Krishnamurthy, A. and A. Vissing-Jorgensen, 2012, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy* 120, 233–267.

[16] Kuong, J. (2015), "Self-Fulling Fire Sales: Fragility of Collateralised Short-term Debt Markets," INSEAD Working Paper.

[17] Martin, A., D. Skeie, and E. von Thadden, 2014, "Repo Runs," *Review of Financial Studies*, forthcoming.

[18] Morris, S. and H. Shin, 2003, "Global games: Theory and applications," in Dewatripont, M., Hansen, M. and S. Turnovsky (Eds), *Advances in economics and econometrics (proceedings of the Eighth World Congress of the Econometric Society)*, Cambridge University Press.

[19] Morrison, E., M. Roe, and H. Sontchi, 2015, "Rolling Back the Repo Safe Harbors," *Business Lawyer*, forthcoming.

[20] Perotti, Enrico, 2014, "The Roots of Shadow Banking," in Claessens S., D. Evanoff, and L. Laeven (Eds), "Shadow Banking Within and Across National Borders," World Scientific.

[21] Quah, J. and B. Strulovici, 2012, "Aggregating the Single Crossing Property, " *Econometrica* 80, 2333–2348.

[22] Stein, J., 2012, "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics* 127, 57–95.