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Coco Design, Risk Shifting Incentives and Financial Fragility

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Coco Design, Risk Shifting Incentives and Financial Fragility

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Abstract

We highlight the ex ante risk-shifting incentives faced by a bank's shareholders/managers when CoCos (contingent convertible capital) are part of the capital structure. The risk shifting incentive arises from the wealth transfers that the shareholders will receive upon the CoCo's conversion under CoCo designs widely used in practice. Specifically we show that for principal writedown and nondilutive equity-converting CoCos, shareholders/managers have an incentive to take on more risk to make conversion more likely because of those wealth transfers. As a consequence, wide spread use of CoCos will increase systemic fragility. We show that such improperly designed CoCos should not be allowed to fill in loss absorption capacity requirements unless accompanied by higher required equity ratios to mitigate the increased risk taking incentives they lead to. Sufficiently dilutive CoCos do not lead to undesired risk taking behavior.

JEL classification: G01, G13, G21, G28, G32

Keywords: Contingent Convertible Capital; Systemic Risk; Risk Shifting Incentives; Capital Requirements

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Nontechnical Summary

CoCos are instruments that start out as debt and then either convert to equity or are written off upon the occurrence of a trigger event. This makes them very attractive to regulators as they avoid a costly bailout by providing loss absorption capacity. However, the design of CoCos may cause bank shareholders to choose actions now that necessitates the loss absorption capacity in the future. Prior to conversion, there is no discernible difference between CoCos and ordinary subordinated debt. However, after conversion, the payoffs to CoCo holders and shareholders are altered drastically, leading to potential wealth transfers from CoCo holders and shareholders. If the shareholders are better off after conversion, then they will choose actions that make conversion more likely. However, these actions have to be decided upon and implemented before anything else occurs. Therefore we examine the expected value of residual equity and determine which action maximizes it.

In our setup, the action is a decision about the riskiness of the assets that the bank invests in. Our base case uses subordinated debt in the capital structure, and our analysis involves replacing the subordinated debt with the same amount of CoCos. The expected value of residual equity with CoCos can be expressed as the value of residual equity with subordinated debt, plus an expected wealth transfer. With this formulation, we can very easily say whether the CoCo induces riskier choices compared to the same amount of subordinated debt – we need only look at what happens to the expected wealth transfer. We find two opposing effects: an increase in risk increases the probability of conversion, but reduces the amount of the wealth transfer. The net effect depends on the type of CoCo issued.

For principal writedown CoCos, the wealth transfer is always from the CoCo holder to the equity holder. For very high levels of risk and/or leverage, the conversion probability factor dominates the wealth transfer factor. We also find that the conversion probability factor is stronger as the writedown percentage increases. This is alarming because it is precisely under those conditions that one wishes the CoCo to be useful. While they may indeed be useful for reducing leverage upon conversion, our analysis shows that the design itself may lead to higher risk levels, without which conversion would not have been necessary.

For equity-converting CoCos, there is a range of values for the dilution parameter such that the wealth transfer goes from the CoCo holder to the equity holder. In such cases, the conversion probability factor always dominates the wealth transfer factor. Therefore these nondilutive CoCos are not very different from principal writedown CoCos. However, dilutive CoCos have wealth transfers from the equity holder to the CoCo holder. In such cases, the conversion probability factor has the same sign as the wealth transfer factor (both negatively related to risk levels), making dilutive CoCos better than subordinated debt.

When banks are faced with private costs of bankruptcy that are quadratic in risk levels, we find that

the risk levels chosen mirror those found earlier: principal writedown CoCos/nondilutive CoCos induce higher risk choices than subordinated debt, which in turn induce higher risk choices than dilutive CoCos. Equity induces the lowest risk choice because it introduces more skin in the game when costs are involved. We also find that for high enough dilution, equity-converting equity may be better than equity itself because of the threat of lower residual value upon conversion.

Finally we analyze the substitutability of CoCos and equity by introducing a social cost of bankruptcy that captures the impact of leverage on the probability of default. In this section, we link risk and leverage: a bank with more leverage has incentives to take on more risk. On the other hand, a regulator who wants to maintain a fixed probability would insist on lower risk for higher leverage. When a bank switches from subordinated debt to the same amount of writedown/nondilutive CoCos, its incentive for risk shifting goes up holding everything else constant. In order to satisfy regulators, the bank must reduce its leverage by issuing more equity, at the expense of deposits. This means these types of CoCos cannot be substitutes for additional equity requirements. On the other hand, when dilutive CoCos are issued, the bank can afford to increase leverage/buy back equity, such that dilutive cocos can substitute for higher equity requirements. This is because while dilutive cocos are not skin in the game in the usual sense, the treat of dilution is credible enough to deter banks from choosing higher risk.

Our results are important because they highlight the need to consider consequences of CoCo design. While increasing total loss absorption capacity is an admirable goal, the method of filling these requirements matter. Not all CoCos are alike in terms of ex ante risk shifting incentives. Therefore, regulators must take care in allowing banks to issue CoCos, such that they don't induce risky behavior that would make the need for loss absorption more likely in the first place.

1   Introduction and literature review

In June 2011, the Basel Committee on Banking Supervision released the final version of Basel III[1]. The document addresses additional measures to ensure the stability of the banking system. Among the new additions are admission of contingent convertible capital (CoCos) as Additional Tier 1 (AT1) (going concern) capital under certain conditions[2]. Failing those conditions, CoCos maybe included as Tier 2 (gone concern, T2) capital instead. CoCos are hybrid instruments issued by banks that start out as debt until a trigger event happens. Broadly speaking, trigger events are either breach of required equity ratios, or reaching a so-called point of non-viability, at the regulator's discretion. Upon the occurrence of either event, CoCos either turn into equity at a pre-specified conversion ratio (or, equivalently a pre-specified conversion price) or are written off partially or completely. They are designed this way in order to relieve the burden of having to raise capital in situations of financial distress (Flannery, 2005) and to spare the taxpayers by increasing the banks' distance to default in distress situations. But the key point of this paper is that many of currently popular CoCo designs imply perverse risk shifting incentives and in that way may actually increase rather than mitigate systemic risk.

Basel provides a framework but has no legal bite in itself, for that the principles need to be embedded in the laws of the countries concerned. The EU has published CRD IV, the EU's directive implementing Basel III, on June 27, 2013 and it has become applicable since January 2014. It will also be the new framework for the single supervisory mechanism. Since CoCos have become part of acceptable AT1 capital under CRD-IV, CoCos have become especially popular among banks faced with more stringent capital requirements in the new banking union. More recently, the Financial Stability Board has released its Total Loss Absorption Capacity (TLAC) Standard[3], worded such that CoCos could be used to fill in the the additional capital requirements. According to Moody's[4], CoCo issuance in 2014 alone amounted to $174 billion. Most of the issuance is by European and Asian banks[5]. Around 60% of those issued to date are of the principal writedown type, where the principal is partially or completely written off when the trigger event happens. The remaining 40% are equity converters where the CoCos convert into equity after the trigger event.

There has been a general consensus in the corporate finance literature that debt acts as a disciplining device, but that it brings its own problems in incomplete information environments. For owners, the threat of bankruptcy carries with it a loss of control rights. On the other hand, limited liability shields equity owners from extreme downside risk, which under limited liability is shifted to creditors, or even to tax

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[1]Basel III: A global regulatory framework for more resilient banks and banking systems
[2]Basel Committee on Banking Supervision (2011)
[3]FSB's TLAC Standard was released on November 9, 2015.
[4]Moody's Quarterly CoCo Monitor - Feb 2015
[5]US banks have not participated in the wave of CoCo issuance because CoCos are treated as equity under US GAAP and as such, do not have tax benefits.

payers through government bail outs. CoCos are designed to reduce these problems by increasing the issuing bank's loss absorption capacity in distress situations. Although CoCos, contrary to for example a share issue, do not raise cash in distress situations, they do increase the distance to default upon conversion. But conversion is not a free pass though. In Chan and van Wijnbergen (2015) we argue that conversion sends a negative signal to depositors about the expected asset returns of the bank and may thereby lead to a higher probability of a bank run in the converting bank, and even in other banks to the extent that they have correlated assets. In this manner CoCo conversion contributes to systemic risk (Chan and van Wijnbergen (2015)). Depending on the CoCo design, significant transfers of wealth may occur between CoCo holders and equity holders. These transfers, and their impact on *ex ante* incentives for managers/shareholders, are the focal point of this paper.

Review of the literature

There is a small but growing body of research on the impact of CoCos on the risk-shifting incentives of banks. Koziol and Lawrenz (2012) only consider equity converter CoCos, and argue that risk-shifting incentives always increase relative to ordinary bonds as long as the old equity holder gets to keep *some* shares after conversion. This strong result depends critically on their assumption that the conversion trigger coincides with the bankruptcy trigger: If asset values decline enough to trigger bankruptcy at a particular leverage ratio, replacing some of the debt by CoCos will obviously leave shareholders better off: with an equal decline in asset values they are left with some claims and bankruptcy is staved off, while in the straight debt case they would have lost everything. We return to their paper in the discussion of our results. Berg and Kaserer (2014) do not present analytical results but numerically simulate the value of equity given an exogenously set mixture of debt and equity converter CoCos for four specific conversion ratios as a function of asset return variance. They argue that risk shifting rises as wealth transfers from CoCo holders to equity holders increase, and observe, like Chan and van Wijnbergen (2015), that the price at which conversion takes place has a direct impact on the magnitude and even sign of these wealth transfers. They also show that several of the existing CoCos such as those issued by Lloyds and Rabobank have prices that *fall* with changes in implied asset volatility, inferring that the market recognizes the risk taken by the banks This finding points at very clear risk taking incentives inherent in the CoCo designs issued by those two banks. Hilscher and Raviv (2014) argue that risk-taking incentives of banks may be mitigated by choosing the conversion ratio properly. For a capital structure containing CoCos, they found conversion ratios such that the resulting equity vega (derivative with respect to asset variance) is equal to zero. This is akin to the suggestion of Calomiris and Herring (2013) on having CoCos which are sufficiently dilutive. On the other

hand, Martynova and Perotti (2015) claim that both convert-to-equity and principal writedown CoCos can mitigate risk-shifting if the trigger level is set properly. In their paper, risk-shifting takes the form of not exerting sufficient effort in monitoring the assets of the bank. However they do not consider the possibility that wealth transfers from CoCo holders to equity holders are affected by the effort (i.e. risk choice) of the manager, nor, more importantly, that the probability of conversion itself is influenced by the manager's effort/risk choice. The latter link is at the core of the analysis presented in this paper.

Chen et al. (2013) do endogenize the conversion[6] in an asset pricing setup similar to Koziol and Lawrenz (2012) and, like them, only consider equity conversion CoCos, but although they derive closed form solutions, they have to use numerical procedures to obtain their results, which as a consequence depend on chosen parameter values. They chose parameter values such that at least some dilution of old shareholders is taking place. As a consequence, conversion in the cases they analyze always imply a loss to old shareholders. But of the more than 200 billion Euro face value CoCos issued so far (December 2015), substantially more than half are issued on terms that actually imply a wealth transfer towards equity holders once conversion takes place, a possibility that plays a substantial role in our paper. In their set up, banks need to continuously roll over debt. This gives rise to rollover costs whenever the market value of the issued debt is lower than the par value of the newly issued debt. The possibility of this happening leads to lower risk shifting by banks, because higher risk increases rollover costs.

Our contribution to the literature is to provide a simple theoretical model of risk-shifting in the presence of CoCos. The simplicity buys us a complete analytical solution, without much loss of generality. We show that many of the results obtained in the literature so far depend on parameter values chosen in the numerical solution approaches, in qualitatively relevant ways. And we apply our framework to the full range of CoCos issued so far: principal writedown CoCos, which are not covered in the academic literature, although they make up some 60% of issues so far by face value, and equity converters but for both dilutive and nondilutive conversion ratios. Risk shifting arises from two forces: the impact of higher risk on the size of the wealth transfers triggered by conversion, and on the probability of that conversion. A key difference of our analysis with the existing literature stems from the explicit attention we pay to that probability of conversion, instead of simply netting out the two often opposing effects. .

We show that for equal loss absorption capacity, all principal write down CoCos and all insufficiently dilutive[7] equity conversion CoCos have substantially worse risk shifting incentives than requiring equity would lead to. Possibly surprisingly, we even show that all principal write down CoCos and insufficiently dilutive equity conversion CoCos have even worse risk shifting incentives than issuing subordinate debt as

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[6]In their continuous time framework, endogenizing conversion comes down to endogenously determining the timing of conversion.
[7]We make the concept "insufficiently dilutive" more precise later in this paper.

an alternative would lead to. As such, there may be an unwarranted buildup of risk within a banking system that extensively uses such risk-shifting CoCos in order to meet regulatory capital or TLAC requirements. The regulatory bodies would seem to be well advised to pay more attention to the risk incentives brought about by the design of CoCos.

2   Setup: on equity, CoCos and embedded options

Limited liability triggers risk transfer between equity and debt. At least since Merton (1974) , it has become customary to model the valuation impact of this risk transfer by incorporating it as a put option written by the creditor and held by the equity holder. Straightforward application of Put-Call parity shows that equivalently equity can be seen as a call option on the firm's assets with the firm's level of debt as strike price. We use a similar approach to incorporate the various risk transfers that take place under different CoCo designs, with as main objective to shed light on the risk taking incentives that different CoCo designs give rise to.

2.1   Equity as a call option

Our model has 3 dates: $t = 0, \ 1, \ 2$. At $t = 0$, the bank obtains its funds from depositors D, subordinated creditors D_s and equity holders e_0 At $t = 0$, all the funds are invested in an asset that has return R'. R' follows a lognormal distribution with parameters (μ, σ^2) for the corresponding normal distribution of $\ln(R')$. The banker can choose the risk level σ of the assets at $t = 0$. At $t = 1$, there is a signal R_1 about the realization of the asset return drawn from the same distribution as the final return at $t = 2$. The creditors of the bank get paid at $t = 2$ after the final return R_2 has materialized. The equity holder of the bank, as the residual claimant, obtains whatever is left after paying off creditors at $t = 2$. Equity holders are protected by limited liability. Call r the safe rate of interest over the periods $0 - 2$. We will assume that the $t = 2$ materialization of asset returns follows instantaneously after the signal at $t = 1$. As stated before, the signal and the final return are drawn form the same distribution.This allows one to write the equity holder's claim as a call option on the asset return with a strike price equal to the face value of the debt of the firm[8]. Assume there is one share with $t = 0$ value e_0 :

$$
\begin{aligned}
e_0 & = \ \exp(-r) \, \mathbb{E} \max[R_2 - D - D_s, \ 0] \\
& = \ \exp(-r) \, \mathbb{E}[R_2 - D - D_s] + P[R, \ D + D_s] \\
& = \ C[R, \ D + D_s] \quad\quad\quad\quad\quad\quad (1)
\end{aligned}
$$

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[8]Merton (1974)

where $C\left[x, \; y\right]$ and $P\left[x, \; y\right]$ are call and put options on x with strike price y, and the transformation from put to call options follow from put-call parity. R is today's expected value of R_2 under the risk neutral distribution.

2.2   CoCos in place of subordinated debt

It is straightforward to understand how the equity of the bank is valued when D_s is subordinated debt (Merton (1974)). It becomes more complicated when the bank issues CoCos instead of subordinated debt. This is because CoCos can convert at $t = 1$ when the signal about asset quality is bad enough. To be more specific, if the signal suggests an asset value below a threshold R_T at $t = 1$, the CoCos can be written off, or convert to equity. The actual terms of conversion are contractually specified, and happens with probability p^c. But this probability is not exogenous.

To analyze p^c, consider the concept of distance to default. If the asset returns are lognormally distributed, and the total face value of the debt amounts to $D + D_s$, we can write the distance to default d_d as

$$d_d \;\; = \;\; \frac{1}{\sigma}\left[\ln\frac{R}{D + D_s} + r - \frac{\sigma^2}{2}\right] \tag{2}$$

where r is the risk-free rate. In our model, if conversion happens, it will only be at $t = 1$, the time the signal is received, so the usual T term takes the value of 1 in this model.

Similarly, we can define a distance to trigger d_T. The trigger event is a signal value that suggests an asset value less than $\rho_T R$ above the debt level $D + D_s$[9]. Then,

$$d_T = \frac{1}{\sigma}\left(\ln\frac{R\left(1 - \rho_T\right)}{D + D_s} + r - \frac{\sigma^2}{2}\right) \tag{3}$$

. With the assumption of lognormally distributed returns, the probability of conversion (which is the same as the probability of hitting the trigger point) is then simply

$$p^c = \Phi\left(-d_T\right) \tag{4}$$

where $\Phi\left(\cdot\right)$ is the cumulative standard normal distribution.

The probability of conversion depends on the trigger level ρ_T and the risk level σ^2 that the managers

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[9]Or equivalently, $\frac{R - D - D_s}{R} < \rho_T \Rightarrow R(1 - \rho_T) < D + D_s$ which leads to Eqn. (3)

choose. We have

$$\frac{\partial p^c}{\partial \sigma} = -\phi(-d_T)\frac{\partial d_T}{\partial \sigma} = \phi(-d_T) \times \left(1 + \frac{d_T}{\sigma}\right) > 0 \tag{5}$$

and

$$\frac{\partial p^c}{\partial \rho_T} = \phi(-d_T)\frac{1}{\sigma(1 - \rho_T)} > 0 \tag{6}$$

Both derivatives are signed intuitively: a higher trigger level increases everything else being the same the probability of the trigger being hit; and similarly shifting more weight out in the tails (increasing the variance) does the same.

2.3   Principal writedown (PWD) CoCos

2.3.1   Full PWD CoCos

Any CoCo conversion can be seen as a reallocation of claims between the CoCo holders and the bank. Consider first a CoCo that is fully written off if $R < R_T$. Upon such a conversion, the CoCo holder sees his investment jump from D_s to 0. On the other hand, the bank's residual equity changes from $C[R,\ D + D_s]$ to $C[R,\ D]$, as D_s is written off.

Before the signal (at $t = 0$), the expected value of a bank's residual equity can be written as

$$\begin{aligned}
e_{pwd} &= (1 - p^c)C\left[R,\ D + D_s | \sigma^2\right] + p^c C\left[R,\ D | \sigma^2\right] \\
&= C\left[R,\ D + D_s | \sigma^2\right] + p^c\left[C\left[R,\ D | \sigma^2\right] - C\left[R,\ D + D_s | \sigma^2\right]\right]
\end{aligned} \tag{7}$$

where p^c is the probability of conversion. As mentioned before, CoCos convert whenever the intermediate signal $R_1 < R_T$.

The term $C\left[R,\ D | \sigma^2\right] - C\left[R,\ D + D_s | \sigma^2\right]$ is the difference between a bank's residual equity when D_s is just subordinated debt and when D_s is a CoCo that has been written off completely. In the case of a full principal write down CoCo, the entire gain accrues to the original equity holders. We refer to this difference as the wealth transfer term W_{pwd}:

$$W_{pwd} = C\left[R,\ D | \sigma^2\right] - C\left[R,\ D + D_s | \sigma^2\right] \tag{8}$$

so we can rewrite Eqn. (7) as

$$e_{pwd} = e_0 + p^c W_{pwd} \tag{9}$$

where $p^c W_{pwd}$ is the expected wealth transfer from a CoCo conversion relative to when the same amount of subordinated debt was issued in its place.

2.3.2 Partial PWD CoCos

Consider now the more general case of a partial PWD CoCo. Let φ represent the fraction of CoCos that remains (so $1 - \varphi$ is written off). The CoCo holder's claim falls from D_s to φD_s. When $\varphi = 1$, the Coco holders do not lose anything upon conversion, so the wealth transfer is 0. As φ moves down from 1 to 0, the CoCo holders lose $(1 - \varphi) D_s$ to the original equity holder. As a result, a PWD CoCo will always result in a loss to CoCo holders, and a corresponding gain (wealth transfer) for equity holders. However, the wealth transfer to the equity holders is not directly $(1 - \varphi) D_s$. When the CoCo converts, the face value of the CoCo falls from D_s to φD_s. This means that the "strike price" of the residual equity also falls from D_s to φD_s, leading to a higher residual equity value. Figure 1 shows the wealth transfer received by equity holders upon the conversion of a PWD CoCo, for values of φ between 0 and 1.

Figure 1: Wealth transfer received by equity holders upon conversion as $\varphi \to 1$
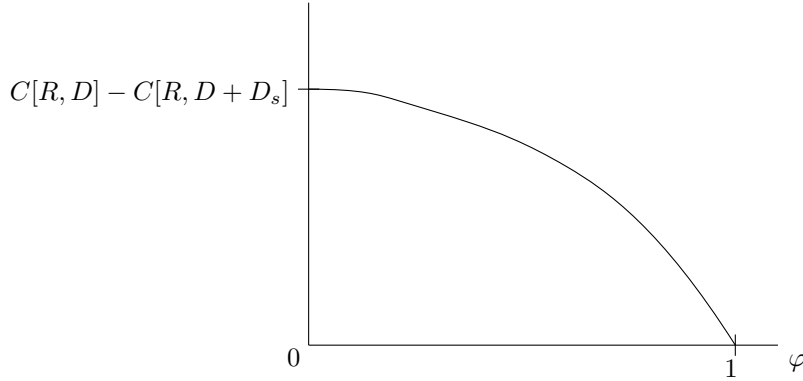
The value of equity, assuming D_s is not subordinated debt but a PWD CoCo with trigger level $\rho_T R$ and write down parameter φ, can be written as e^{φ}_{pwd}

$$
\begin{aligned}
e^{\varphi}_{pwd} &= (1 - p^c)\, C\,[R,\, D + D_s] + p^c\, C\,[R,\, D + \varphi D_s] \\
&= C\,[R,\, D + D_s] + p^c\,[C\,[R,\, D + \varphi D_s] - C\,[R,\, D + D_s]] \\
&= e_0 + p^c W_{pwd}(\varphi) \tag{10}
\end{aligned}
$$

10

with the following expression for the wealth transfer:

$$W_{pwd}(\varphi) \quad = \quad C\left[R,\ D + \varphi D_s\right] - C\left[R,\ D + D_s\right] \tag{11}$$

Note that the full writedown case is merely a special case of the partial writedown, with $\varphi = 0$. In Figure 1, this is the point with coordinates $(0,\ C\left[R,\ D\right] - C\left[R,\ D + D_s\right])$. Obviously for given ex ante risk choice the conversion probability does not depend on φ[10]. Clearly a CoCo that does not write down anything after conversion (i.e. $\varphi = 1$) is equivalent to subordinated debt:

$$\begin{aligned} W_{pwd}(\varphi = 1) \quad &= \quad C\left[R,\ D + D_s\right] - C\left[R,\ D + D_s\right] \\ &= \quad 0 \end{aligned} \tag{12}$$

2.4 Convert-to-Equity (CE) CoCos

An alternative design lets the CoCo convert into new equity, partially diluting the old shareholders. Upon conversion of such CoCos, liabilities worth D_s disappear from the balance sheet, such that the total residual equity becomes $C\left[R,\ D\right]$. However, even if CoCos convert into new equity, there may still be wealth transfers from the CoCo holders to the original shareholders. . This depends on the conversion rate ψ (or equivalently, the share price $1/\psi$). If no conversion takes place, the value of the CoCo remains at D_s. Upon conversion at rate ψ, the CoCo holder loses D_s worth of debt but gains ψD_s shares, entitling them to a $\psi D_s/1+\psi D_s$ share in residual equity, where the existing shareholders' share is normalized to 1.

As mentioned previously, the losses sustained by CoCo holders are gains for equity holders. However, while conversion of CE CoCos change the value of residual equity from $C\left[R,\ D + D_s\right]$ to $C\left[R,\ D\right]$, it also means that equity holders must share the gain with the new shareholders (former CoCo holders). Figure 2 illustrates how the wealth transfer from CoCo holders to the existing shareholders vary with the dilution parameter ψ.

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[10]but φ has an impact on ex ante risk choice, as we show below.

Figure 2: Wealth transfer received by equity holders upon conversion as $\psi \to \infty$
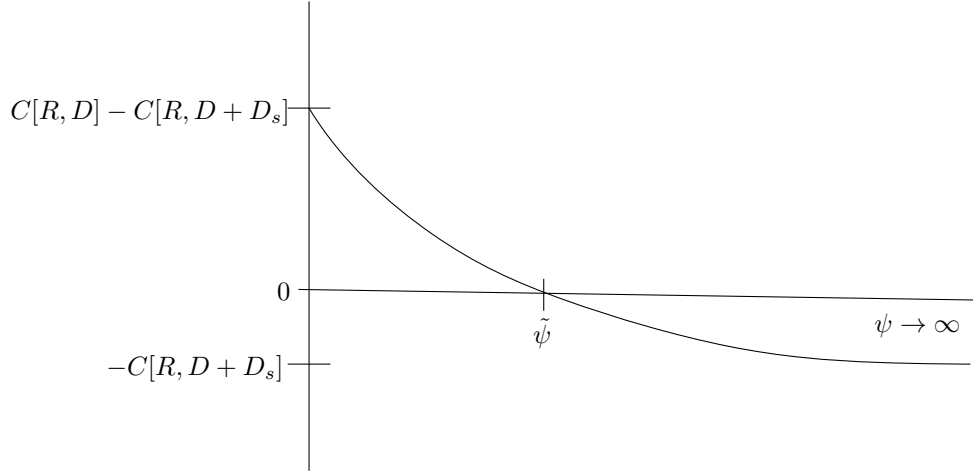
The ex ante equity value of the existing shareholders taking into account the possibility of conversion and subsequent dilution then is:

$$
\begin{aligned}
e_{ce} &= (1 - p^c)C\left[R,\ D + D_s | \sigma^2\right] + p^c\left(\frac{1}{1 + \psi D_s}\right)C\left[R,\ D | \sigma^2\right] \\
&= e_0 + p^c\left(\frac{1}{1 + \psi D_s}C\left[R,\ D | \sigma^2\right] - C\left[R,\ D + D_s | \sigma^2\right]\right)
\end{aligned}
\tag{13}
$$

where the existing claim of the original shareholders is normalized to 1. This expression is similar to the full PWD case, except for the scale factor $1/1 + \psi D_s$. This scale factor represents the reduced share of the original shareholders on residual equity, due to the emergence of ψD_s new shares from the conversion.

Note that for $\psi = 0$, the expression is equal to the expression for a 100% PWD CoCo, where the CoCo holder gets nothing after conversion

$$
e_{ce}(\psi = 0) = e_{pwd}(\varphi = 0)
\tag{14}
$$

. This can be seen in Figure 2 as the point with coordinates $(0,\ C\left[R,\ D + \varphi D_s\right] - C\left[R,\ D + D_s\right])$. Note the similarity with Figure 1. As $\psi \to \infty$, complete dilution occurs after conversion. In such a case, the original shareholder loses his claim of $C\left[R,\ D + D_s\right]$. This is illustrated in Figure 2 as indicated by $\psi \to 0$.

Clearly the wealth transfer that will take place when conversion occurs depends on the degree of dilution and can take either sign: positive if there is little dilution (low ψ or high conversion price $\frac{1}{\psi}$) and negative when there is substantial dilution (high ψ or low conversion price $\frac{1}{\psi}$):

$$
W_{ce} = \frac{1}{1 + \psi D_s}C\left[R,\ D | \sigma^2\right] - C\left[R,\ D + D_s | \sigma^2\right]
\tag{15}
$$

We can therefore define a benchmark conversion rate (or, equivalently, a benchmark conversion price equal to the inverse of the benchmark conversion rate) $\psi = \tilde{\psi}$, with $\tilde{\psi}$ defined such that the wealth transfer is zero[11] ($W_{ce} = 0$):

$$\tilde{\psi} \quad = \quad \frac{1}{D_s} \left\{ \frac{C\left[R,\ D|\sigma^2\right] - C\left[R,\ D + D_s|\sigma^2\right]}{C\left[R,\ D + D_s|\sigma^2\right]} \right\} \tag{16}$$

. In Figure 2, $\tilde{\psi}$ is where the wealth transfer line crosses the ψ axis. The number of new shares $\tilde{\psi}D_s$ valued at the pre-conversion value of $C\left[R, D + D_s|\sigma^2\right]$ just equals the total change in residual equity post conversion: $C\left[R,\ D|\sigma^2\right] - C\left[R,\ D + D_s|\sigma^2\right]$[12]. If the conversion rate is higher (the price lower), than the benchmark value, the wealth transfer goes from old equity holders to CoCo holders/new equity holders. And for a lower conversion rate (higher conversion price), the wealth transfer flows from CoCo holders to equity holders. Eqn. (17) lists the extreme cases:

$$\begin{aligned} W_{ce}(\psi = 0) &= W_{pwd}(\varphi = 0) \\ W_{ce}(\psi = \tilde{\psi}) &= 0 \\ W_{ce}(\psi = \infty) &= -e_0 \end{aligned} \tag{17}$$

For $\psi = 0$, the CE CoCo is equivalent to a (full) PWD ($\varphi = 0$); for $\psi = \infty$ the CoCo holder gains the full value of the pre-conversion equity e_0 and at $\tilde{\psi}$ the conversion is neutral in the sense of implying a zero wealth transfer[13].

3   Risk shifting incentives and CoCo design

Consider next the incentives for ex ante risk shifting under various capital structures, ex ante in the sense that the risk choice σ occurs before the signal that might possibly trigger conversion is received[14]. Analysis under alternative capital structures have been used by several authors for different reasons: Berg and Kaserer (2014); Chen et al. (2013); Hilscher and Raviv (2014) use this framework to assess risk-shifting incentives and the probability of default, Albul et al. (2013) use them to determine the impact on a bank's market value. Zeng (2014) takes it a step further by combining both risk decisions of the bank and the op-

---

[11]We find this by setting Eqn. (15) to 0 and solving for ψ.

[12]cf Calomiris and Herring (2013) for a similar discussion and the recommendation to use a conversion price closely related to our definition of $\tilde{\psi}$.

[13]This price is critical according to Sundaresan and Wang (2015) if multiple equilibria are to be avoided in the case of market-based (share price) conversion triggers.

[14]We interpret risk shifting as a choice for riskier assets by the Bank's trading department. Alternatively one can think of lower investment in risk management.

timal capital structure when CoCos are in the mix. Different from these papers, we use alternative capital structures in order to form benchmarks for expected wealth transfers. The key issue is that any decision on the riskiness of the underlying investment project affects both the probability of a conversion and the final outcome, because both are affected by elements that are drawn from the same distribution. Initially we will simply establish the marginal value of increasing σ for different capital structures, and in particular for different CoCo designs. In Section 4 we introduce bankruptcy costs that vary with risk levels, similar to Kashyap and Stein (2004). We examine the trade off between increasing the probability of obtaining the wealth transfer against increasing bankruptcy costs under different capital structures. In Section 5 we examine the substitutability of CoCos for capital requirements.

There are of course risk-shifting incentives because of leverage when D_s consists of subordinated debt, these are captured by the call option representation of equity we introduced at the beginning of Section 2, and the derivative of e_0 with respect to σ. Since we are interested in comparing risk-shifting incentives under a given CoCo design with those under a subordinated debt issue[15], we investigate the impact of changes in σ on the differences $e_{pwd} - e_0$ and $e_{ce} - e_0$, which are the expected wealth transfers. To make sure that we analyze a pure risk effect not mixed up with increases in wealth, we structure the increase in risk in such a way that the mean of R stays unchanged (i.e. a mean-preserving spread in variance). Since R is drawn from a lognormal distribution, this requires an offsetting downward shift in $\mu : R$ is lognormal, so $\ln R \sim N\left(\mu, \sigma^2\right)$. Therefore $\mathbb{E}\left(R\right) = \exp\left(\mu + \frac{\sigma^2}{2}\right) \Rightarrow d\mu = -\sigma d\sigma$.

3.1 Principal write down (PWD) CoCos and risk shifting incentives

Little has been written about PWD CoCos. Among the papers that mention these are Berg and Kaserer (2014) and Martynova and Perotti (2015). Berg and Kaserer (2014) considers 100% writedown CoCos, termed as "convert to steal" and show using numerical methods that this type of CoCo increases banks' incentives to increase risk. On the other hand, Martynova and Perotti (2015) opine that full writedown CoCos reduce risk shifting incentives, as the fall in debt due to conversion effectively raises the bank's skin in the game. In this section though, we show that if one makes the analysis prior to conversion, the risk shifting incentives for such CoCos are actually higher, because we not only look at the actual fall in debt from conversion, but we also look at the impact of the risk choice on the probability of conversion.

---

[15]For the equity holder it does not matter whether D_s is subordinated or not since in both cases equity is junior to all debt.

3.1.1 Risk taking incentives for given write down parameter φ

Consider then the risk shifting incentive in the presence of PWD CoCos, where we define the risk-shifting incentive (RSI) as the derivative of the expected wealth transfer $(p^c W_{pwd} = e_{pwd} - e_0)$ with respect to σ:

$$
\begin{aligned}
RSI_{pwd}(\varphi) &= \frac{\partial (e_{pwd} - e_0)}{\partial \sigma} \\
&= \frac{\partial p^c \left(C\left[R, D + \varphi D_s | \sigma^2\right] - C\left[R, D + D_s | \sigma^2\right] \right)}{\partial \sigma} \\
&= \underbrace{\frac{\partial p^c}{\partial \sigma} \left(C\left[R, D + \varphi D_s | \sigma^2\right] - C\left[R, D + D_s | \sigma^2\right] \right)}_{CF_{pwd}} + \underbrace{p^c \frac{\partial C\left[R, D + \varphi D_s | \sigma^2\right] - C\left[R, D + D_s | \sigma^2\right]}{\partial \sigma}}_{WF_{pwd}}
\end{aligned}
\tag{18}
$$

Eqn. (18) shows that the derivative of the RSI has two components. The conversion probability factor (CF_{pwd}) represents the increase in the probability of conversion as risk increases, holding the wealth transfer term constant. From Eqn. (5) we can easily see that $\frac{\partial p^c}{\partial \sigma} > 0$, so CF_{pwd} is always larger than zero. An increase in the risk level reduces the distance to the trigger and so brings the wealth transfer closer. In the literature on the incentive effects of CoCo design, this probability has mostly been taken as exogenous (see for example Martynova and Perotti (2015)); but Eqn. (18) shows that doing so misses out on a potentially perverse component of the incentive structure created by the use of CoCos instead of straight equity: raising the probability of conversion makes it more likely that the equity holder obtains the wealth transfer implicit in conversion. For PWD CoCos this wealth transfer is always positive by design, so the CF is always positive too.

The wealth transfer factor (WF_{pwd}) (the second term in Eqn. (18)) represents the impact of the increase in the risk level on the value of the wealth transfer itself, holding the probability of conversion constant. The value of the wealth transfer springs from the disappearance of part of the debt. While the value of the wealth transfer itself is positive, WF_{pwd} shows it is decreasing in the risk taken. To see why, note that WF_{pwd} can be written as the difference between the vegas[16] of two call options that differ only in the strike price (i.e. the outstanding debt). This allows us to write WF_{pwd} in terms of a cross-derivative with respect to the strike price. With lognormally distributed asset returns R and $t = 1$ as before, and for a general strike price K, this cross derivative (which we call V_K) can be simply derived from the analytical expression for vega:

$$
V\left[R, K | \sigma^2\right] = \frac{\partial C\left[R, K | \sigma^2\right]}{\partial \sigma} = R\phi(d_1) > 0
\tag{19}
$$

with ϕ the probability density function of the standard normal and with $d_1 = \frac{1}{\sigma}\left[\ln \frac{R}{D + D_s} + r + \frac{\sigma^2}{2}\right]$, the expected value[17] of the log difference of the stock price minus exercise price K. Using the mean value

---

[16] Vega is the derivative of a call option with respect to volatility.

[17] more precisely the expected value conditional on the stock price being high enough to trigger exercise.

theorem, we can express WF_{pwd} as a function of the cross derivative of C with respect to σ and the strike price :

$$
\begin{aligned}
WF_{pwd} &= p^c \left(V \left[R, \ D + \varphi D_s | \sigma^2 \right] - V \left[R, \ D + D_s | \sigma^2 \right] \right) \quad (20) \\
&= -p^c \left((1 - \varphi) D_s V_K \left[R, \ D' | \sigma^2 \right] \right)
\end{aligned}
$$

where $D \in [D + \varphi D_s, \ D + D_s]$. So the WF_{pwd} term is directly proportional to a second derivative of the embedded call options and as such is of second order, i.e. zero in a linear approximation of the option value, compared to the first order risk shifting impact through the probability of conversion (our CF_{pwd}). In Annex 1 we show that the vega of a call option always increases with the strike price, so $\partial C (R, D + \varphi D_s) / \partial \sigma$ is lower than $\partial C (R, D + D_s) / \partial \sigma$ and WF_{pwd}, the impact of higher risk levels on the size of the wealth transfer, is negative.

Thus, summing up, PWD CoCos risk shifting incentives have two components. The conversion probability factor is a positive first order term which indicates that any wealth transfer is more easily obtained by increasing the risk levels and so increasing the probability of conversion. We also have the wealth transfer factor, a negative second order term which states that doing so also decreases the wealth transfer that takes place upon conversion. These two effects work against each other. From the analytical expression given in Annex 1 we can see in which regions the positive and the negative terms dominate respectively: in particular, the second order term is small for a sufficiently high leverage $\frac{D'}{R}$, for a sufficiently high variance parameter σ, and interestingly, for a low probability of conversion p^c. This implies that exactly when fragility is high (high leverage and/or high volatility of asset returns), PWD CoCos imply perverse risk taking incentives, because then the CF_{pwd} term dominates. The negative effect operating through the impact of higher risk levels on the size of the wealth transfer (conditional on conversion) only plays a dominant role when leverage and/or variance are sufficiently low. But the higher the leverage of the banks swapping part of their subordinated debt into PWD CoCos, the more risk incentives worsen when that subordinated debt is replaced by a CoCo. This is quite a strong result since CoCos count as AT1 capital[18] and thus are considered close to equity and more equity obviously reduces risk taking incentives; nevertheless swapping out of subordinated debt into PWD CoCos leads to higher risk taking incentives if leverage and/or volatility are sufficiently high, i.e. exactly when we worry about it most.

An interesting observation concerns the impact of the trigger ratio ρ_T. A higher trigger ratio leads to a lower distance to conversion d_T and thus increases the probability of conversion p^c. Since WF_{pwd} is

---

[18]Under Basel III and the European directive CRD-IV implementing Basel III, all types of CoCo are considered "AT1", Additional Tier1 capital if the trigger level exceeds a specified level and if regulators can force conversion if in their opinion a Point Of Non-Viability is reached.

proportional to p^c, raising the trigger level increases the negative WF_{pwd} term. Also, from Eqn. (5) one can see that a higher trigger level and associated lower distance to conversion lowers the impact of a change in variance on the conversion probability and thus lowers the positive CF_{pwd} term. So higher trigger levels reduce the incentive to increase risk for any given PWD CoCo, and therefore will make PWD CoCos less dangerous.

Eqns. (18) and (20) immediately show a result that is maybe trivial but serves as a check: for $\varphi = 1$, the expression for RSI becomes zero. The CF_{pwd} term equals zero because the wealth transfer equals zero for a zero writedown, and the WF_{pwd} term is proportional to $(1 - \varphi)$ and therefore also equals zero for $\varphi = 1$, so we get $RSI(\varphi = 1) = 0$. This is logical since RSI measures the relative risk taking incentive with respect to the case where the CoCo is replaced by subordinated debt, but for $\varphi = 1$ the PWD CoCo is in fact equivalent to subordinated debt, so in that case the relative risk taking incentive cannot be anything but zero. For lower values of φ the analysis turns more complex, we turn to that analysis below.

3.1.2 Risk taking incentives as a function of the write down parameter φ

The impact of φ on the risk-shifting incentives likewise depends on whether we are in a high or in a low risk environment (i.e. on whether leverage and/or σ are high or low). Of course for $\varphi = 1$ (no writedowns), the PWD CoCos are equivalent to subordinated debt and the risk taking incentives as we measure them (with respect to the case of subordinated debt) will be zero. But for $\varphi < 1$, the terms CF_{pwd} and WF_{pwd} again play their different roles. Since p^c does not depend on φ, the derivative of our indicator for relative risk taking incentives, the expected wealth transfer $p^c W_{pwd}$, with respect to φ equals:

$$
\begin{aligned}
&\frac{\partial}{\partial \varphi}\left[\frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma}\right] \\
&= \underbrace{-\frac{\partial p^c}{\partial \sigma} \exp\left(-r\right) \Phi\left(d_2\left(D + \varphi D_s\right)\right) D_s}_{\partial CF_{pwd}/\partial \varphi} + \underbrace{p^c \phi\left(d_1\left(D + \varphi D_s\right)\right) D_s \frac{R}{D + \varphi D_s} \frac{d_1}{\sigma}}_{\partial WF_{pwd}/\partial \varphi}
\end{aligned}
\tag{21}
$$

where the notations $d_1\left(D + \varphi D_s\right)$ and $d_2\left(D + \varphi D_s\right)$ refer to d_1 and d_2 with strike price $D + \varphi D_s$. The derivative of the CF_{pwd} term with respect to φ is unambiguously negative: since φ is the fraction of the debt retained, we unambiguously have that a smaller writedown leads to lower risk shifting incentives through the first order CF_{pwd} term measuring the impact. Thus, the larger the writedown fraction, the higher the first order risk shifting incentive is.

Consider now the second term in Eqn. (21), the derivative of the WF_{pwd} term with respect to φ. This expression is clearly always positive: a higher writedown percentage (lower φ) strengthens the offsetting effect through the impact of σ on the size of the wealth transfer, and thus tends to work in the opposite

direction of the first order impact. However, we show in Annex 1 that as σ goes up, $\frac{\phi(d_1)d_1}{\sigma}$ tends to zero. Thus it follows from Eqn. (21) that the derivative with respect to φ of WF_{pwd} will be lower for higher σ so for σ sufficiently high the derivative of CF_{pwd} will dominate. So we once again get that the first order effect dominates in circumstances of high leverage and high risk. Thus in situations where one expects CoCos to be useful (high risk and high leverage), it turns out that their presence increases risk shifting.

3.2 Convert-to-Equity CoCos (CE) and risk shifting incentives

The risk shifting analysis is easily extended to CE CoCos. If D_s is replaced by CE CoCos, the CoCo is replaced by ψD_s new shares, so the old equity holders are diluted and now own only a fraction $\frac{1}{1+\psi D_s}$ of the equity. Thus upon conversion, the expected wealth transfer is:

$$p^c W_{ce} = p^c \left\{ \frac{1}{1+\psi D_s} C[R,\ D] - C[R,\ D+D_s] \right\} \tag{22}$$

As mentioned in Section 3, when $\psi = \tilde{\psi}$, the wealth transfers are zero for CE CoCos, so that the CE CoCos are equivalent to subordinated debt[19]. When $\psi = 0$, the wealth transfer is at its highest level as the original shareholders do not have to share with the CoCo holders.

The risk shifting incentive beyond the subordinated debt component is given by the derivative of $p^c W_{ce}$ (equivalent to $e_{ce} - e_0$) with respect to σ, with μ reduced correspondingly to preserve the mean value R:

$$
\begin{aligned}
RSI_{ce} &= \frac{\partial}{\partial \sigma} \left\{ \frac{p^c}{1+\psi D_s} C[R,\ D] \right\} - \frac{\partial}{\partial \sigma} \left\{ p^c C[R,\ D+D_s] \right\} \\
&= \underbrace{\frac{\partial p^c}{\partial \sigma} \left[\frac{C[R,\ D]}{1+\psi D_s} - C[R,\ D+D_s] \right]}_{CF_{ce}} + \underbrace{p^c \left[\frac{V[R,\ D]}{1+\psi D_s} - V[R,\ D+D_s] \right]}_{WF_{ce}}
\end{aligned} \tag{23}
$$

RSI_{ce} is written in a similar manner as RSI_{pwd}, where we also refer to the components as conversion probability factor (CF_{ce}) and wealth transfer factor (WF_{ce}). CF_{ce} is directly proportional to the wealth transfer and depends on the first derivative of p^c with respect to σ, while WF_{ce} is proportional to the second derivative of the call option value $C[R,\ K]$ for strike price K, and in that sense of second order while CF_{ce} is of first order. As in the previous subsection, CF_{ce} is positive while WF_{ce} is negative. However, as argued in the previous section, as ψ moves closer to 0, for σ high enough, CF_{ce} remains positive while WF_{ce} goes to zero. So at $\psi = 0$, $RSI_{ce}(\psi = 0) > 0$, meaning that there are risk-shifting incentives for convert-to-equity CoCos where the conversion rate is 0 - that is, they are even worse than if we had subordinated debt instead of the CoCos.

---

[19] PWD CoCos are equivalent for the limiting case $\varphi = 1$.

Consider now the case when $\psi = \infty$. In such a case, when CoCos convert, the CoCo holders obtain control of the entire equity base because the new shares issued to them completely dilutes the shares held by existing shareholders. Then, we would have

$$RSI_{ce}\,(\psi = \infty) \quad = \quad \underbrace{\frac{\partial p^c}{\partial \sigma}\,[-C\,[R,\,D + D_s]]}_{CF_{ce}} + \underbrace{p^c\,[-V\,[R,\,D + D_s]]}_{WF_{ce}} \tag{24}$$

Notice that the wealth transfer term becomes negative due to the disappearance of the call option value when CoCos have converted. In this case, because the value of a call option is positive, CF_{ce} is clearly negative! The vega of a call option is positive as well, so WF_{ce} is also negative. Thus, $RSI_{ce}\,(\psi = \infty)$ has unambiguously negative risk shifting incentives. In other words, for highly dilutive CoCos, an increase in the risk taken only makes the expected wealth transfer negative. Thus, CoCos may be a good disciplinary tool for reducing risk-shifting of existing shareholders if sufficiently dilutive. But can we make "sufficient dilution" more precise?

Since $RSI_{ce}(\psi = 0) > 0 > RSI_{ce}(\psi = \infty)$, we get by continuity a crossing at zero for a positive ψ, although that zero point does not necessarily occur at $\tilde{\psi}$, a term we have introduced before as the value of ψ that sets the wealth transfer equal to zero. To find this zero point, we return to the expression for the risk shifting incentive from Eqn. (23):

$$RSI_{ce} \quad = \quad \underbrace{\frac{\partial p^c}{\partial \sigma}\left[\frac{C\,[R,\,D]}{1 + \psi D_s} - C\,[R,\,D + D_s]\right]}_{CF_{ce}} + \underbrace{p^c\left[\frac{V\,[R,\,D]}{1 + \psi D_s} - V\,[R,\,D + D_s]\right]}_{WF_{ce}}$$

At $\psi = \tilde{\psi}$, CF_{ce} is 0 but WF_{ce} is negative. This means that the threshold level of ψ that makes the risk shifting incentive exactly zero lies somewhere within the interval $\left[0,\,\tilde{\psi}\right]$. Because CF_{ce} and WF_{ce} are generally of opposite signs, we need only choose a ψ that makes CF_{ce} positive and exactly offsets the negative value of WF_{ce}. In other words, choose ψ such that $p^c\left[\frac{V[R,\,D]}{1 + \psi D_s} - V\,[R,\,D + D_s]\right] = \frac{\partial p^c}{\partial \sigma}\left[\frac{C[R,\,D]}{1 + \psi D_s} - C\,[R,\,D + D_s]\right]$. Let us call this value $\psi_{RSI = 0}$ [20]where

$$\psi_{RSI=0} \quad < \quad \psi_{WT=0} = \tilde{\psi} \tag{25}$$

. Thus, any $\psi \in [0,\,\psi_{RSI=0})$ will yield a positive risk shifting incentive (i.e. worse than in the alternative capital structure with subordinated debt instead of CoCos). Any $\psi \in [\psi_{RSI=0},\,\infty)$ makes the risk-shifting incentives negative, regardless of the risk-level taken, i.e. for all values of σ.

---

[20]The results are consistent with those of Hilscher and Raviv (2014), who find the conversion ratio that achieves zero vega. However, they only consider the wealth transfer and the leverage channels. Our calculations for the conversion ratio also takes the endogenous probability of conversion into account.

These results are stronger than the ones obtained for the case of PWD CoCos since they do not rely on second order terms being small enough (leverage and/or risk taken high enough). The reason is that a change in the conversion price does not affect the total transfer from creditors to old and new equity holders, but only the allocation of that transfer over old and new shareholders. So a higher or lower share price does not change the leverage post-conversion. But a change in the write down parameter affects the size of the transfer directly and will have an impact on post-conversion leverage and from there on ex ante risk shifting incentives. A lower φ (higher write down percentage) makes the wealth transfer conversion brings to old equity holders bigger and thus increases risk shifting incentives. But it simultaneously leads to lower post-conversion leverage, which has the opposite effect. This latter channel is absent in the case of equity converters.

4   Risk choices of banks under different capital structures

After sketching the link between CoCo design, capital structure and marginal relative risk taking incentives, the natural next step is to investigate the actual choice that is likely to be made and how they relate to socially optimal choices. To that end we introduce private (and indirectly) social costs of bankruptcy. Private actors (in our case the bank managers/equity holders) respond to the (expected) private costs of bankruptcy, but regulators set the restrictions under which private actors have to operate, and do so incorporating the social costs of bankruptcy. The costs of bankruptcy *conditional on bankruptcy having occurred* are are kept exogenous to our analysis, in line with the partial equilibrium set up of the entire paper, call them X. Note that these costs refer to (expected) bankruptcy costs other than foregone asset returns since those are captured in the various call option terms. One should think of reputation or legal costs when the bank defaults on its senior debt holders (i.e. on D)[21]. But the probability of bankruptcy occurring (p^D) is directly related to the risk choices and leverage decisions made by banks and regulators and so is endogenously determined in our model. We next approximate the expected costs of bankruptcy by a first order Taylor series approximation:

$$
\begin{aligned}
p^d X &= p^d(\sigma,\ D)X \\
&\approx \frac{1}{2}b\sigma^2 + cD
\end{aligned}
\tag{26}
$$

absorbing X in the coefficients of the Taylor series approximation and omitting the irrelevant zero order constant.

---

[21] For a recent survey of the economics of bankruptcy focusing specifically on banks, see Marinč and Vlahu (2011)

Furthermore, regulators cannot control risk levels chosen directly since σ is not directly observable for outsiders; but regulators can insist on a maximum leverage ratio (set a minimum capital to asset ratio) acting as a subsequent constraint on the choices to be made by private banks/equity holders in banks. This implies that we model the interaction between banks and the regulator as a Stackelberg game, where the regulator sets the minimum capital ratio (in the context of our model with given asset total, this is equivalent to a limit on D) and the bank, acting as a follower, chooses σ given D. The regulator, in choosing D, incorporates what the bank will do when confronted with the minimum leverage ratio implied by D. So we assume in this section that regulators have imposed a total loss absorption capacity (TLAC), or, equivalently, a leverage ratio $(R - D)/R$. Subject to that TLAC requirement, set by the regulator, we then ask the question how capital structure influences the optimal risk choice. We return to the regulator's choice of maximum leverage D in the next section. We furthermore assume that the minimum capital ratio set by the regulator is in fact a binding constraint and analyze private choices given the regulator's choice of leverage D, in the next section we take the next step and explicitly consider the interaction between the regulator's choice for a given capital asset ratio $1 - D$ and the private risk choice σ.

So when managers/equity holders of a bank decide on their risk levels, their objective function consists of the value of residual equity modeled here as the call option value on the firm's assets with senior debt D as a strike price, as before, minus expected bankruptcy costs.:

$$\max C\left[R,\ D\right] - \frac{1}{2}b\sigma^2 \tag{27}$$

Since in this entire Section 4 all optimization is conditional on a given leverage ratio, we leave out the term cD for notational convenience. Kashyap and Stein (2004) adopt a similar characterization of the expected costs of bankruptcy.

Other papers also focus more on the probability of bankruptcy than on the probability of conversion (Hilscher and Raviv (2014); Chen et al. (2013)), perhaps due to their emphasis on the loss-absorption capacity of CoCos, while our focus is much more on ex ante shareholder behavior, and the role the probability of conversion plays therein, with its associated wealth transfers. In (Hilscher and Raviv (2014); Chen et al. (2013)), the probability of bankruptcy is influenced by the asset level that leads to bankruptcy, which is chosen endogenously by shareholders in their analysis, but the leverage and capital structure choice and their interaction with risk choices are not considered explicitly in these papers.

## 4.1   When TLAC is met through equity: D_s is equity

Consider first the case where the full TLAC requirement is met through equity, i.e. D_s fully consists of equity. Equity holders need to weigh the gains in residual equity value versus the higher expected costs of bankruptcy $\frac{1}{2}\sigma^2 b$ that a higher risk choice also brings. When D_s is equity, the equity holder's problem is, as in Eqn. (27),

$$\max C\left[R,\ D\right] - \frac{1}{2}b\sigma^2$$

because the strike price equals D when D_s is equity. This leads to the first order condition describing the optimal risk choice as

$$V\left[R,\ D\right]\quad =\quad b\sigma \tag{28}$$

$V\left[R,\ D\right]$ captures the marginal benefit from incurring an additional unit of risk σ. We show in Annex 5 that the marginal benefit is falling as σ increases. σb captures the marginal cost of incurring an additional unit of risk, and is increasing in σ. Figure 3 illustrates the marginal costs and benefits of increasing risk under the conditions outlined in Annex 5.

Figure 3: σ_e^* vs σ_s^*
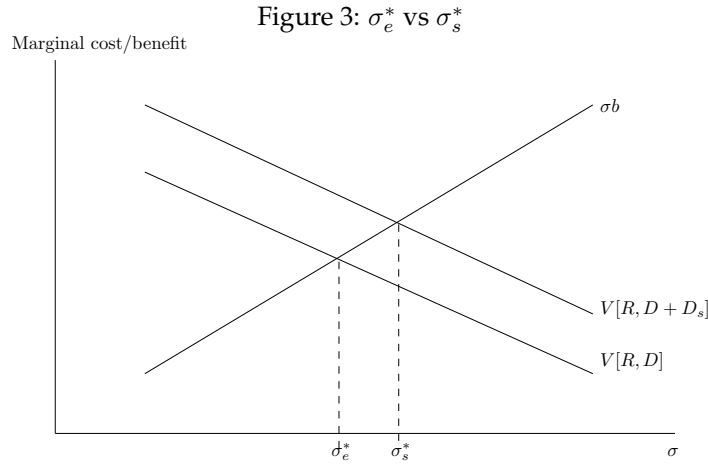
Note that the $V\left[R,\ D\right]$ line crosses the σb line in Figure 3. So the first order condition in Eqn. (28) is satisfied for some σ_e^*. And we show in Annex 5 also that the objective function is concave for sufficiently large marginal bankruptcy costs b, high volatility level σ and/or sufficiently low expected asset value R (see Annex 5 for a more precise characterization of the concavity conditions). We assume them to be satisfied so (28) indeed characterizes a maximum as a function of the leverage ratio $R/(R-D)$.

4.2 When TLAC requirements are (partially) met through subordinated debt: D_s is subordinated debt

When D_s is subordinated debt (T2 capital in Basel III terms), the equity holder's risk choice problem is

$$\max C\left[R, \ D + D_s\right] - \frac{1}{2}b\sigma^2 \tag{29}$$

because when D_s is subordinated debt, the strike price is $D + D_s$: some of the lower asset returns at $t = 2$ are absorbed by the subordinated debt holders as the equity holder is protected by limited liability. The corresponding first order condition is:

$$V\left[R, \ D + D_s\right] \ = \ b\sigma \tag{30}$$

Note that the derivative of vega with respect to strike price K, V_K, is strictly positive[22]. This means that the $V\left[R, \ D + D_s\right]$ line should lie above the $V\left[R, \ D\right]$ line for any given σ. Figure 3 illustrates the higher risk incentives for having D_s subordinated debt relative to none. As a result, for the same cost line σb, the σ that satisfies the first order condition is higher for D_s subordinated debt than for the same amount of additional equity.

We can write (30) approximately as:

$$V(R, \ D|\sigma_e) + V_\sigma(\sigma_s - \sigma_e) + V_K D_s \ = \ b(\sigma_s - \sigma_e) + b\sigma_e \tag{31}$$

$$\Rightarrow$$

$$\sigma_s \ = \ \sigma_e + \frac{V_K D_s}{b - V_\sigma} > \sigma_e$$

So banks which meet part of their TLAC requirements through subordinated debt have higher risk-shifting incentives than banks that meet all of the TLAC requirements through equity.

4.3 When TLAC requirements are (partially) met through PWD CoCos

Consider now the introduction of CoCos, in particular assume subordinated debt D_s is replaced by CoCos of equal face value. We first consider the case of principal write down (PWD) CoCos with write down percentage φ: the equity holder maximizes

$$C\left[R, \ D + D_s\right] + p^c W_{pwd} - \frac{1}{2}b\sigma^2 \tag{32}$$

---

[22]See Annex 1 for the derivation showing this.

which differs by the expected wealth transfer $p^c W_{pwd}$ from the case when D_s is subordinated debt. The probability of default and the associated expected costs of default are the same as in the case when D_s is subordinated debt since default will only occur when the total loss absorption capacity inclusive of subordinated debt has been eaten up. But as with subordinated debt this only happens when D is defaulted on so we always have $p^c > p^D$. Maximizing Eqn. (32) leads to the first order condition:

$$V[R,\ D+D_s] + \frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma} \quad = \quad b\sigma \tag{33}$$

Eqn. (18) and surrounding text show that $\frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma} > 0$ whenever σ is large enough or when leverage is high enough. Thus, the graph of $V[R,\ D+D_s] + \frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma}$ lies above that of $V[R,\ D+D_s]$. Figure 4 illustrates the relationship of the two graphs in high leverage/high risk circumstances.

Figure 4: σ_s^* vs σ_{pwd}^*
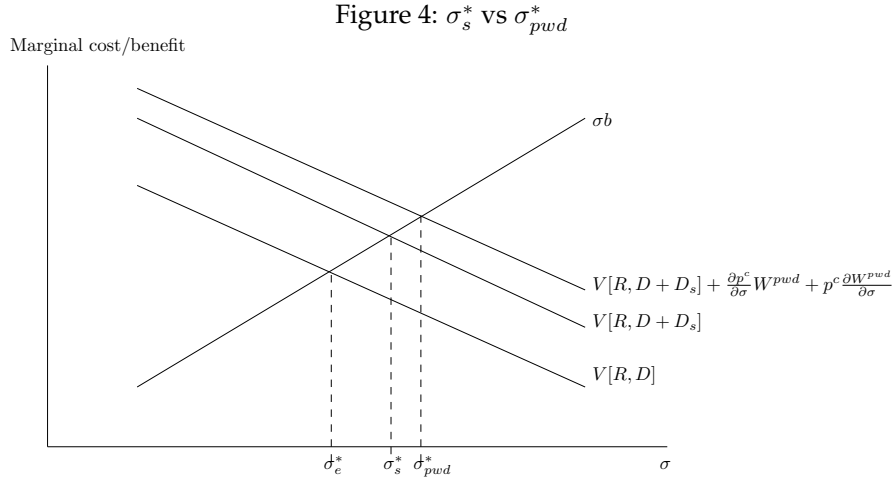
As long as the σb line is unchanged, the σ that satisfies Eqn. (33) is higher than the one that satisfies Eqn. (30). Using Eqn. (30), we can find an expression for this σ:

$$V[R,\ D+D_s] + \frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma} \quad = \quad b(\sigma_{pwd} - \sigma_s) + b\sigma_s$$

$$\Rightarrow$$

$$\sigma_{pwd} \quad = \quad \sigma_s + 34 \frac{\frac{\partial p^c}{\partial \sigma} W_{pwd} + p^c \frac{\partial W_{pwd}}{\partial \sigma}}{b} > \sigma_s \tag{34}$$

Thus, the risk level σ_{pwd}^* that satisfies the first order condition in Eqn. (33) is higher than the risk level that solves the first order condition pertaining to when D_s is subordinated debt: $\sigma_{pwd}^* > \sigma_s^*$.

From a risk-shifting standpoint, it is clear that having D_s as principal writedown CoCos is worse than having them as subordinated debt, at least in a high leverage-high risk environment where we have $\frac{\partial p^c}{\partial \sigma} W_{pwd} +$

$p^c \frac{\partial W_{pwd}}{\partial \sigma} > 0$. Principal Write Down CoCos meet the criteria for inclusion in AT1 capital because they reduce leverage *after* conversion, providing an additional buffer to the bank. However, while they increase the buffer ex post, they also encourage risk-shifting behavior ex ante, making it more likely that the buffer will in fact be necessary at some future date. Even although they are classified as AT1, they induce worse risk shifting behavior from equity holders than even subordinated debt does, and much worse than what requiring true equity would induce.

4.4 When TLAC requirements are (partially) met through convert-to-equity CoCos

When D_s is a convert-to-equity CoCo, the equity holder's maximization problem is

$$\max C\left[R,\ D + D_s\right] + p^c W_{ce} - \frac{1}{2} b\sigma^2 \tag{35}$$

which once again differs by $p^c W_{ce}$ compared to the maximand when D_s is subordinated debt. Maximizing (35) leads to the first order condition

$$V\left[R,\ D + D_s\right] + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} = b\sigma \tag{36}$$

From Eqn. (23) we know that:

$$\frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} = p^c \left[\frac{V\left[R,\ D\right]}{1 + \psi D_s} - V\left[R,\ D + D_s\right] \right] + \frac{\partial p^c}{\partial \sigma} \left[\frac{C\left[R,\ D\right]}{1 + \psi D_s} - C\left[R,\ D + D_s\right] \right]$$

From the definition of the threshold degree of dilution as defined in Eqn. (52), we know that when the convert-to-equity CoCos are sufficiently dilutive (i.e. when $\psi > \psi_{RSI=0}$), we get $\frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} < 0$ and vice versa. For those values of $\psi > \psi_{RSI=0}$, the graph of $V\left[R,\ D + D_s\right] + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma}$ always lies below the graph of $V\left[R,\ D + D_s\right]$ when plotted against σ. Also, the marginal cost function here, σb, is the same one as when D_s is subordinated debt. Figure 5 illustrates the case for when $\psi \in (\psi_{RSI=0},\ \infty)$.

Figure 5: σ_s^* vs σ_{ce}^* for $\psi \in (\psi_{RSI=0}, \infty)$
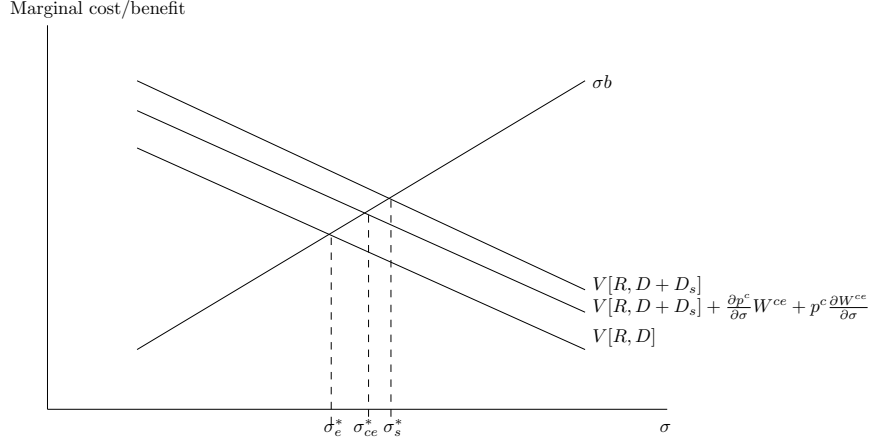
This means that using σ_s^* would not satisfy the first order condition set out in Eqn. (36), because such a σ would result to the left hand side being lower than the right hand side. Because vega is decreasing in σ, we need some $\sigma_{ce}^* < \sigma_s^*$ to fulfill Eqn. (36). By rewriting Eqn. (36) in a manner similar to Eqn. (33), we can find an expression for this σ:

$$V[R,\ D + D_s] + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} \quad = \quad b(\sigma_{ce} - \sigma_s) + b\sigma_s \tag{37}$$

$$\sigma_{ce} \quad = \quad \sigma_s + \frac{\frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma}}{b}$$

Thus for $\psi > \psi_{RSI=0}$, $\sigma_{ce} < \sigma_s$. Thus, we have that the risk levels chosen by banks can be ranked as follows: $\sigma_{ce}^* < \sigma_s^* < \sigma_{pwd}^*$[23].

It is then clear that sufficiently dilutive convert-to-equity CoCos induce better risk choices than the same amount of subordinated debt. As such, their inclusion as AT1 capital is an improvement, though as we will argue in the next subsection, not as good as true equity. While convert-to-equity CoCo is not skin in the game ex ante like equity is, it serves a similar purpose in the reduction of risk-shifting, because it reduces residual equity value after conversion, thereby making it an effective deterrent to risk.

4.5 How does a CE CoCo compare to equity?

Thus far we have proven two sets of results, with the second inequality holding if CoCos are sufficiently dilutive:

$$\sigma_e^* < \sigma_s^* < \sigma_{pwd}^* \qquad \text{and} \qquad \sigma_{ce}^* < \sigma_s^* < \sigma_{pwd}^*$$

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[23]Of course $\psi < \psi_{RSI=0} \Rightarrow \sigma_{ce} > \sigma_s$, such that the ranking of risk levels chosen by banks become $\sigma_s^* < \sigma_{ce}^* < \sigma_{pwd}^*$

But can we determine how convert-to-equity CoCos compare with straight equity in terms of risk choice? We need only to compare risk-shifting incentives between these two cases, since they offer equal loss absorption capacity. Recall from Eqn. (28) that when D_s is equity, the first order condition is

$$V[R, \ D] = b\sigma$$

as D_s does not form part of the strike price.

From Eqn. (36), for the case when D_s is a convert-to-equity CoCo, the first order condition is

$$V[R, \ D + D_s] + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} = b\sigma$$

where if we decompose $V[R, \ D + D_s]$ in the manner of Eqn. (31), we can rewrite the first order condition as

$$V(R, \ D|\sigma_e) + V_\sigma(\sigma_{ce} - \sigma_e) + V_K D_s + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} = b(\sigma_{ce} - \sigma_e) + b\sigma_e$$

$$\Rightarrow$$

$$\sigma_{ce} = \sigma_e + \frac{V_K D_s + \frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma}}{b - V_\sigma} \quad (38)$$

where from Eqn. (23),

$$\frac{\partial p^c}{\partial \sigma} W_{ce} + p^c \frac{\partial W_{ce}}{\partial \sigma} = p^c \left[\frac{V[R, \ D]}{1 + \psi D_s} - V[R, \ D + D_s] \right] + \frac{\partial p^c}{\partial \sigma} \left[\frac{C[R, \ D]}{1 + \psi D_s} - C[R, \ D + D_s] \right]$$

We assume that the marginal bankruptcy cost coefficient b is high enough for the necessary condition for concavity of the objective function, $b - V_\sigma > 0$, to hold[24]. Thus, any ψ that sets $V_K D_s + \frac{\partial p^c}{\partial \sigma} W^{ce} + p^c \frac{\partial W^{ce}}{\partial \sigma} \geq 0$ makes the risk shifting incentive of D_s convert-to-equity CoCo smaller than or equal to the risk-shifting incentive for D_s equity, for equal TLAC. In particular, it is

$$\psi \geq \frac{1}{D_s} \left\{ \frac{p^c V[R, \ D] + \frac{\partial p^c}{\partial \sigma} C[R, \ D]}{p^c V[R, \ D + D_s] + \frac{\partial p^c}{\partial \sigma} C[R, \ D + D_s] - \left(\frac{R\phi(d_1)}{K} \right) \left(\frac{d_1}{\sigma} \right) D_s} - 1 \right\} \quad (39)$$

Call the ψ for which Eqn. (39) holds with equality ψ_{eq}. Note that ψ_{eq} resembles $\psi_{RSI=0}$ in Eqn. (52).

[24]In Annex 5 we show that for sufficiently low leverage and/or sufficiently high variance σ^2, $V_\sigma < 0$ in which case this condition holds for any $b > 0$.

However, $\psi_{eq} > \psi_{RSI=0}$ because

$$\frac{\partial p^c}{\partial \sigma} C\left[R,\ D + D_s\right] + p^c V\left[R,\ D + D_s\right] > \frac{\partial p^c}{\partial \sigma} C\left[R,\ D + D_s\right] + p^c V\left[R,\ D + D_s\right] - \left(\frac{R\phi\left(d_1\right)}{K}\right)\left(\frac{d_1}{\sigma}\right) D_s$$

for large enough σ, where $p^c V\left[R,\ D + D_s\right]$ forms part of the denominator of Eqn. (52).

This means that if the conversion ratio ψ of CE CoCos are sufficiently dilutive (i.e. when $\psi \in [\psi_{ce},\ \infty)$), the CE CoCos are better than straight equity in terms of risk-shifting incentives. Figure 6 illustrates the relationship between the risk shifting line for equity and for CE CoCos with varying dilution parameters. .

Figure 6: How a CE CoCo compares to equity
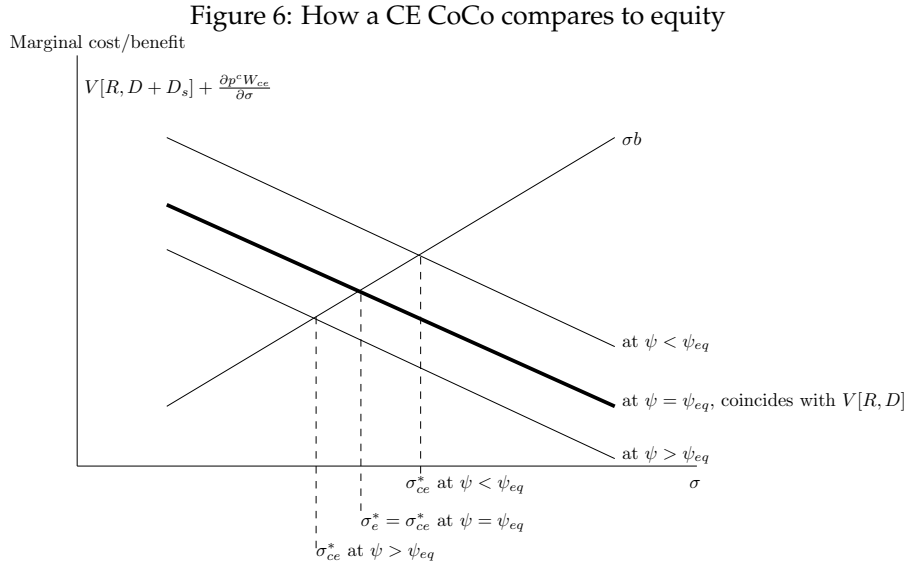
Thus we have the following results: for $\psi \in [0, \psi_{RSI=0}]$, we have

$$\sigma_e^* < \sigma_s^* < \sigma_{ce}^*$$

For $\psi \in [\psi_{RSI=0},\ \psi_{eq}]$, on the other hand, we have

$$\sigma_e^* < \sigma_{ce}^* < \sigma_s^* < \sigma_{pwd}^*$$

. Finally, for $\psi \in [\psi_{eq},\ \infty]$, we get a strong result:

$$\sigma_{ce}^* < \sigma_e^* < \sigma_s^* < \sigma_{pwd}^*$$

So when the CoCo is sufficiently dilutive (i.e. $\psi > \psi_{eq}$), D_s CE CoCos are even better than the same amount of straight equity in that they provide less of a risk shifting incentive for equal TLAC! And even

28

when they are insufficiently dilutive for this strong result, but still provide at least a zero wealth transfer to old equity holders, they still perform better than either subordinated debt or PWD CoCos, in that they provide less risk shifting incentives for the same TLAC as subordinated debt would. But if the CoCos are in fact not dilutive at all, i.e. conversion would actually benefit shareholders by providing them with a wealth transfer at conversion, they are worse than subordinated debt in that they provide even worse risk shifting incentives for equal Total Loss Absorption Capacity. In that case they clearly should not be part of AT1 capital.

4.6 The trigger level ρ_T

The value of residual equity when CoCos are in the capital structure for CE CoCos equals

$$E_{coco} \quad = \quad C\left[R,\, D + D_s\right] + p^c \left\{ \frac{C\left[R,\, D\right]}{1 + \psi D_s} - C\left[R,\, D + D_s\right] \right\} \tag{40}$$

. The trigger level has no impact on the face value D_s of the CoCo prior to conversion, nor on the wealth transfer that occurs upon conversion. The trigger level ρ_T only appears indirectly, in that it influences the probability of conversion p^c, a higher trigger level makes conversion more likely (cf Eqn. 6). Setting ρ_T higher makes the expected value of residual equity with CoCos higher:

$$\frac{\partial E_{coco}}{\partial \rho_T} \quad = \quad \frac{\partial p^c}{\partial \rho_T} \left\{ \frac{C\left[R,\, D\right]}{1 + \psi D_s} - C\left[R,\, D + D_s\right] \right\} \tag{41}$$

which brings the wealth transfer closer upon first glance. So raising the trigger level is good or bad for equity holders depending on whether the wealth transfer goes to them or to the CoCo holder:

$$\frac{\partial E_{coco}}{\partial \rho_T} \quad \lesseqgtr \quad 0 \Leftrightarrow \psi \lesseqgtr \widetilde{\psi} \tag{42}$$

When the wealth transfer is positive, an increase in the trigger level makes the value of residual equity higher, ceteris paribus. On the other hand, the impact of a higher ρ_T on risk shifting incentive is different: for a generalized CE CoCo, the risk shifting incentive is

$$\frac{\partial E_{coco}}{\partial \sigma} \quad = \quad V\left[R,\, D\right] + \underbrace{p^c \frac{\partial W_{ce}}{\partial \sigma}}_{<0} + \underbrace{\frac{\partial p^c}{\partial \sigma} W_{ce}} \tag{43}$$

where $\frac{\partial p^c}{\partial \sigma} W_{ce} > 0$ for $\psi \in \left(0\ \tilde{\psi}\right)$ and negative otherwise.

The impact of increasing the trigger level ρ_T on the risk-shifting incentive is

$$\frac{\partial^2 E_{coco}}{\partial\sigma\partial\rho_T} = \underbrace{\frac{\partial p^c}{\partial\sigma}\frac{\partial W_{ce}}{\partial\sigma}}_{<0} + \underbrace{\frac{\partial^2 p^c}{\partial\sigma\partial\rho_T}W_{ce}}_{<0} \tag{44}$$

for a nondilutive CoCo. The same can be said about PWD CoCos, as the wealth transfer is always positive in those instances. So raising the trigger level ρ_T always reduces the risk shifting incentives embedded in the CoCo design provided that the wealth transfer is positive, because $\frac{\partial^2 p^c}{\partial\sigma\partial\rho_T} < 0$. This is a possible way of mitigating the ill effects of CoCos that were designed to favor the original shareholders. This result supports the BIS requirement of a sufficiently high trigger level[25] for the CoCo to qualify as AT1 capital. As for highly dilutive CE CoCos, the fact that $\frac{\partial^2 p^c}{\partial\sigma\partial\rho_T} < 0$ interacts with the negativity of the wealth transfer, such that the net effect is more ambiguous[26].

5 CoCos, privately and socially optimal risk taking and capital requirements

The goal of capital regulation is to protect the financial system from bankruptcy externalities (Kashyap and Stein (2004)), and the taxpayer from the need to bail out bankrupt banks in a crisis. The idea is to increase the distance to default, by imposing capital requirements the financial system is provided a cushion, a minimum degree of loss absorption capacity. Since we leave these externalities (or, relatedly, the cost of public funds) external to our model, we assume the Government, when trading off the social costs of bankruptcy with its desire to interfere with financial intermediation as little as possible, arrives at a target probability of default which is not to be exceeded

$$p^d(\sigma,\,D) \;=\; \overline{p^d} \tag{45}$$

The Basel II and Basel III capital requirements against (risk weighted) assets are in fact derived from the requirement that the probability of default be kept below a certain number; see VanHoose (2007) for a very instructive survey of the history of and literature on the impact of capital requirements[27].

In striving for a probability of default below a target level, the system not only seeks to provide a minimum level of loss absorption capacity for given asset side portfolio structure, it also aims to reduce balance

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[25]According to the Basel III FAQ, the trigger level should be 5.125% or higher.

[26]Martynova and Perotti (2015) also find that increasing the trigger level induces the banks to exert more effort in order to stave off conversion. This is consistent with our result that risk shifting incentives decline as the trigger level rises.

[27]We do not model a direct link between the risk choice σ and the required equity ratio $1 - D$, thereby implicitly assuming that changes made in risk choice do not move the assets concerned outside their initial Basel III risk class .

sheet risk by discouraging banks from making socially undesirable asset side portfolio choices through higher capital requirements: in a straight debt-equity capital structure, higher capital requirements will reduce the value of the implicit put options equity holders receive, from creditors through limited liability and/or from tax payers through explicit or implicit bailout guarantees. But we have shown in the previous section that a substantial presence of CoCos on the liability side of the balance sheet considerably changes the risk taking incentives banks face on the margin for given capital requirements. It is therefore natural to ask the question to what extent CoCos, for given loss absorption capacity, influence the risk choice reducing impact of capital requirements, and whether the answer to that question should have consequences for the regulatory treatment of CoCos. In particular, if a regulator has set a given equity ratio $1 - D$ assuming the loss absorption capacity is provided by either equity or subordinated debt (we will look at both cases), should they adjust that ratio when these more traditional forms of (T1 and T2) capital are replaced by Co-Cos[28]? In analyzing the interplay between regulator and bank, it is natural to assume a Stackelberg game structure: the regulator, as a Stackelberg leader, sets capital requirements knowing how banks as Stackelberg followers will respond to that particular requirement. We therefore start by outlining the risk choices a bank makes given the capital requirement set by the regulator, and then determine, given that reaction curve, how the regulator sets that requirement so as to achieve its desired probability of default $\overline{p^d}$.

In Section 4, we have shown that there is a positive relationship between a bank's leverage and choice of risk levels. As a result, we can draw a risk curve (RC) that shows the bank's best risk choice given a certain debt level. Figure 7 illustrates this relationship.

Figure 7: Bank's risk curve against regulator's chosen probability of default

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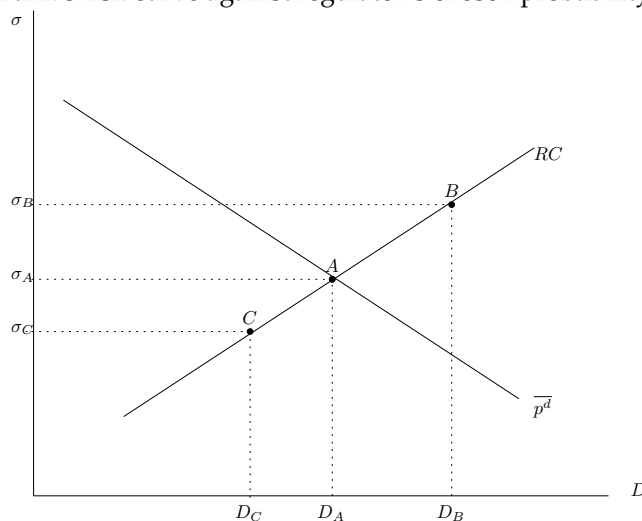
[28]VanHoose (2007) argues in a similar vein that while models focusing on asset side portfolio choice are "broadly supportive" of the risk reducing impact of higher capital requirements, taking other banking functions and/or the complete bank balance sheet into account lead to a much less clear cut picture. See in particular Diamond and Rajan (2000).

RC can be interpreted as the reaction of the Stackelberg follower. At issue then is how the Stackelberg leader, the Central Bank acting as regulator, picks the right point off that curve by setting a required ratio of equity to assets, or equivalently in our set up, the maximum senior debt level D. As mentioned earlier, regulators are interested in limiting bankruptcy externalities while also minimizing interference in financial intermediation and in making that trade off arrive at a target probability of default $\overline{p^d}$. The actual level of the probability of default is a function of risk σ and leverage D. To maintain any given level of $\overline{p^d}$, any increase in risk must be compensated for by a decrease in leverage, or in other words, an increase in capital requirements. This set of compensating variations folllows from partially differentiating Eqn. (45):

$$b\,\partial\sigma + c\,\partial D \;=\; 0 \tag{46}$$
$$\left.\frac{\partial\sigma}{\partial D}\right|_{\overline{p^d}} \;=\; -\frac{c}{b} < 0$$

where c and b are the coefficients of the Taylor approximation to Eqn. (46) given in Eqn. (26). The downward sloping line labeled $\overline{p^d}$ in Figure 7 illustrates the tradeoff between risk and leverage that this choice of a given default probability implies. A higher (lower) maximum allowed default probability corresponds to an upward (downward) shift in the downward sloping line in Figure 7.

Effectively, since σ is not contractible, the regulator has only one tool at his disposal to achieve its target $\overline{p^d}$, the power to impose capital requirements, given the reaction curve of the bank. The bank will then choose the risk level that corresponds to the capital requirements imposed by the regulator in a manner indicated by the line RC. The RC line derives from partially differentiating the first order condition of the bank's optimization problem in Eqn. (28):

$$\left.\frac{\partial\sigma}{\partial D}\right|_{RC} \;=\; \frac{1}{b}V_D > 0$$

For instance, in Figure 7, if the regulator chooses leverage D_B, the bank will choose risk level σ_B, which is higher than what the regulator wishes (corresponds to a higher probability of default since Point B is above the $\overline{p^d}$ line. On the other hand, if the regulator chooses leverage D_C, the bank will choose risk level σ_C, which is now much lower than what the regulator wishes (Point C corresponds to a lower default probability since C is below the $\overline{p^d}$ line. Only if the regulator imposes leverage D_A will the bank choose a risk level σ_A that is compatible with the $\overline{p^d}$ specified by the regulator, at the intersection of the $\overline{p^d}$and RC lines: A is the equilibrium solution to the Stackelberg game between regulator and commercial bank. Consider now the introduction of CoCos. Suppose that the capital requirement $1 - D$, is met with D_s

subordinated debt; $1 - D - D_s$ is therefore equity. Consider now what happens when, possibly in response to tightening of capital standards, subordinated debt is replaced by CoCos.

Principal Write Down or insufficiently dilutive CE CoCos

We saw in the previous section that complete PWD CoCos have even worse risk shifting incentives than subordinated debt (and thus substantially worse than straight equity). So given that subordinated debt only qualifies as T2 capital under Basel III, it is arguable that complete PWD CoCos should probably never have been included as AT1 equity to begin with, whatever the trigger level; if subordinated debt only qualified as T2 capital, and PWD CoCos provide the same loss absorption capacity but even worse risk taking incentives, the logic of letting them qualify as (A)T1 capital is altogether unclear. Because conversion of a writedown CoCo wipes out a junior creditor before reaching equity and so allows the equity holder/manager in effect to jump the seniority ladder, they will not act in a safer manner even when compared with the case where these instruments are subordinated debt instead. Much of the recent (2013-2015) flood of CoCos issues has done just that, replace expiring subordinated debt. What that should lead to in our framework is shown in Figure 8 below. Because the switch of subordinated debt for PWD CoCos increases risk taking incentives (cf Eqn. (34)), the RC curve shifts up to RC'. In fact the risk shifting incentives are higher for both PWD and nondilutive CE CoCos relative to subordinated debt because of the positive expected wealth transfer that will occur upon conversion and that is made more likely by riskier behavior. The largest shift in the risk curve is brought about by changing D_s to either a full PWD CoCo ($\varphi = 0$) or a CE CoCo with dilution parameter $\psi = 0$, as these CoCos provide the largest amounts of wealth transfers.

Figure 8: Upward shift in the risk curve due to replacing subordinated debt by risk-inducing CoCos
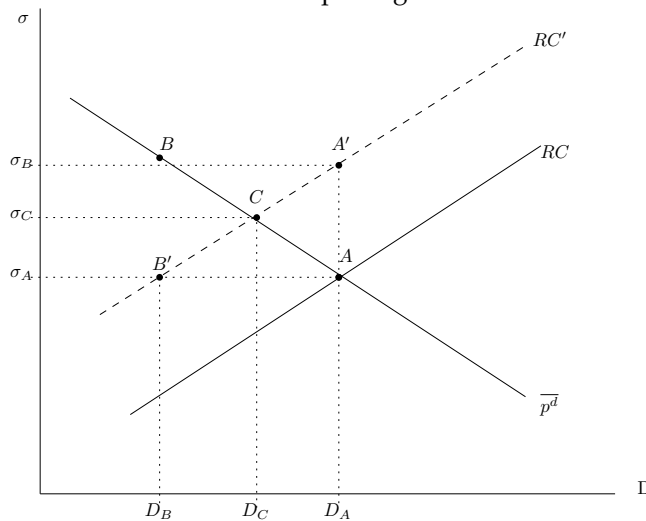
So suppose that the regulator has chosen the probability of default $\overline{p^d}$, and the system has settled at a combination of σ and D consistent with $\overline{p^d}$, i.e. Point A in Figure 8. Then D_s subordinated debt is completely

replaced with either a PWD or a nondilutive CE CoCo. This change causes the shift from RC to RC'. As the bank did not change its leverage ratio, it still has D_A leverage, but because of the potential wealth transfer brought about by the change from subordinated debt to equity, the risk incentives are higher: the bank's position is now at Point A', where leverage is at D_A but risk choice is at $\sigma_B > \sigma_A$. What should the regulator do in this situation? At Point A', the risk level σ_B and leverage D_A combination implies a probability of default which is higher than $\overline{p^d}$. To get back at $\overline{p^d}$ he should impose higher capital requirements D_B, as indicated in 8 . But raising capital requirements by an additional $D_A - D_B$ in turn leads to a lower risk choice of σ_A, which now implies a probability of default below $\overline{p^d}$, and so on. The new set of equilibrium values is at Point C, with a higher risk choice than at Point A but a correspondingly larger loss absorption capacity because of the associated higher capital requirement.

Formally this can be seen as follows: in order to stay along the RC line, any change in risk level must be accompanied by a change in the leverage as well. The equation

$$\partial \sigma - \alpha \partial D \;\; = \;\; 0 \tag{47}$$

captures this principle, where α is a constant and 0 is a normalization. A shift upwards from RC to RC' keeps the slope fixed but changes the intercept by $\sigma_{A \to B} = \sigma_B - \sigma_A$. Thus, the equation describing the RC' line is

$$\partial \sigma - \alpha \partial D \;\; = \;\; \sigma_{A \to B} \tag{48}$$

Also to stay along the $\overline{p^d}$ line, the equation

$$b \partial \sigma + c \partial D \;\; = \;\; 0 \tag{49}$$

must be satisfied. In order to find the $(\sigma, \; D)$ combination that satisfies both Eqns. (48) and (49), we simply solve the following system of equations:

$$\begin{bmatrix} 1 & -\alpha \\ b & c \end{bmatrix} \begin{bmatrix} \sigma_{A \to C} \\ -\Delta_{A \to C} \end{bmatrix} = \begin{bmatrix} \sigma_{A \to B} \\ 0 \end{bmatrix}$$

where we relabel $\partial\sigma$ as $\sigma_{A\to C} = \sigma_A - \sigma_C$ and ∂D as $-\Delta_{A\to C} = D_C - D_A$. The system yields

$$
\begin{bmatrix} \sigma_{A\to C} \\ \Delta_{A\to C} \end{bmatrix} = \begin{bmatrix} \frac{c\sigma_{A\to B}}{c+b\alpha} \\ \frac{-c\sigma_{A\to B}}{c+b\alpha} \end{bmatrix}
$$

which shows that an increase in risk must be accompanied by lower leverage requirement (higher capital requirements). Lower leverage increases the bank's "skin in the game", which offsets the higher risk incentives put forth by PWD or insufficiently dilutive CoCos and their expected wealth transfers. To summarize, the issuance of PWD and nondilutive CE CoCos to fulfill TLAC requirements causes banks to choose higher risk levels than would obtain if straight equity or even subordinated debt would have been chosen, and the regulator should impose correspondingly higher capital requirements. PWD and nondilutive CE CoCos therefore are poor substitutes for equity for compliance with TLAC requirements.

Sufficiently dilutive CoCos

We have illustrated the case for the PWD and nondilutive CE CoCos. The situation is better when dilutive CE CoCos are considered, because the movement of the expected wealth transfer is away from the equity holders towards to CoCo holders. As the dilution parameter moves from $\psi = 0$ to $\psi = \psi_{RSI=0}$, the upward shift in RC becomes smaller. At $\psi = \psi_{RSI=0}$, the risk incentives do not change anymore such that the RC line does not shift. As for highly dilutive CoCos, where $\psi < \psi_{RSI=0}$, RC shifts downwards instead of upwards. Figure 9 shows this other case. Seen this way, dilutive CoCos are a legitimate (A)T1 component.

Figure 9: Downward shift of the risk curve due to replacing subordinated debt by dilutive CoCos
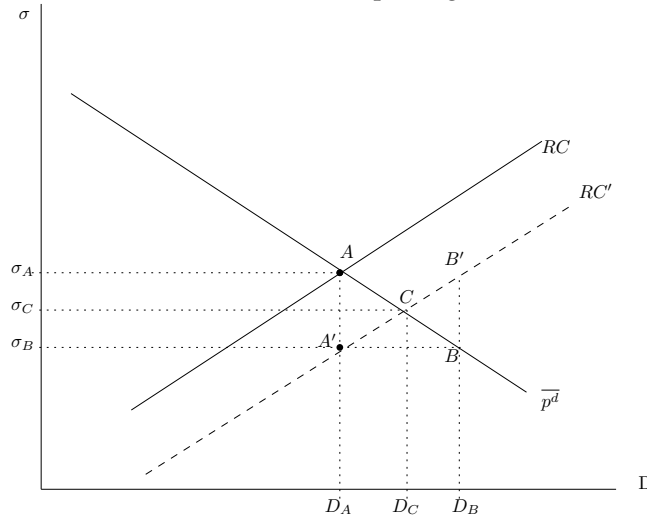
6   Discussion and conclusions

CoCos have become popular among bankers since the emergence of Basel III. And with the Financial Stability Board's release of the TLAC[29] Standard, CoCo issuances are expected to continue further as banks are required to increase loss absorption capacity to roughly double Basel III levels, but can do so entirely by issuing CoCos without any further restriction except a sufficiently high trigger value. However, CoCos are not without problems. In an earlier paper, we have shown that conversion of the CoCos of a particular bank will increase systemic risk through the signalling effect of such a conversion Chan and van Wijnbergen (2015). Also, while pricing has been extensively discussed in the literature, little is known about the empirical performance of the pricing methods proposed; in particular, are tail (conversion) risks actually priced in? Also little is known about their impact on bank behavior as a function of their design. It is that latter question we address in this paper.

CoCos, whether of the principal writedown or convert-to-equity variety, will convert with some probability p^c. As such, the ex ante residual value of a bank that has CoCos in the capital structure has to take these probabilities into account. With probability $1 - p^c$, the CoCo will not convert, thus maintaining its subordinated debt status. With probability p^c, the CoCo converts, raising the residual value of the bank by reducing its leverage, which is exactly what has convinced regulators to accept their role in meeting capital (and TLAC) requirements. However, the higher residual equity value may under many CoCo designs issued in practice accrue to the old shareholders, thereby partially or completely wiping out the CoCo holder before equity is reached, in spite of the CoCo holders status as junior creditor. The literature has paid attention to the wealth transfers taking place upon conversion (cf Chan and van Wijnbergen (2015) for an extensive discussion), but has largely modelled the conversion probability as exogenous. But doing so implies missing out on the crucial role that the shareholders/managers have on influencing this probability through their risk choices. This matters because of the perverse wealth transfers that many CoCo designs used in practice imply and plays a key role in this paper.

We define wealth transfers from the old (i.e. prior to conversion) shareholders' point of view - that is the differences in residual equity that are triggered by a conversion induced reduction in leverage. When calculating residual equity this way (which we do using call option valuation as in Merton (1974)), subordinated debt and unconverted CoCos of the same amount are equivalent because both senior to equity. In the same way, there is no difference between equity and converted CoCos of the same amount, at least to the extent that the newly created equity value accrues to the old equity holders. This fact enables us to write the ex ante residual value of a CoCo-issuing bank as a weighted average of the respective residual values with

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[29]Total Loss Absorption Capacity

subordinated debt, and with equity, with the conversion probability as the weight on the latter, and one minus that probability as the weight on the former. Equivalently, this approach allowed us to decompose this value as the residual value with subordinated debt, plus an expected wealth transfer term. The expected wealth transfer is the product of the conversion probability and the wealth transfer term.

A number of things emerge from this analysis: first, the risk decision of the shareholder affects the weights used in the calculation of the ex ante residual value of equity. Effectively, this allows the shareholder to manipulate the ex ante residual value to his/her advantage and may lead to additional risk taking when CoCos replace other sources of funding. Put another way, the risk decision allows the shareholder to make his desired event (whether it is conversion or not) more likely. But second, and to some extent counteracting the effect, higher levels of risk in fact reduce the wealth transfer conditional on conversion, although we can show the second effect is second order compared to the first effect. But in general it is therefore not enough to focus only on the probability of wealth transfers taking place when the risk level changes: the overall impact of CoCos on risk shifting incentives is the net effect of two opposing forces: the rise in the conversion probability, and the fall in the wealth transfer. We analytically derive conditions under which the first effect dominates the second. This matters in particular when the wealth transfer is to the advantage of the equity holder, since in that case his risk shifting incentives will increase when the first effect dominates the second. Unfortunately that happens when initial risk levels or leverage ratio's are sufficiently high, in a precisely defined manner, exactly the circumstances that should alarm regulators.

The strength of the risk-shifting incentives is strongly influenced by CoCo design. We show that PWD CoCos always transfer wealth to equity holders upon conversion when the risk level chosen or the bank's leverage is high enough. This is because the fall in the wealth transfer conditional on conversion brought about by higher risk levels is more than offset by the increased probability that conversion will take place under such circumstances.

Moreover, the risk shifting incentives becomes worse as the writedown parameter increases. This is because a higher write down percentage leads to a higher wealth transfer to the old equity holders upon conversion, thereby giving him/her a larger interest in such a conversion actually taking place. The larger interest increases the risk-taking incentives. As a result, whenever PWD CoCos are in the capital structure, the risk levels chosen by the banks are higher than what they would have chosen under the same amount of subordinated debt. This is an alarming result, and maybe also unexpected, given that CoCos are seen as a mix of debt and equity. Thus principal write down CoCos do not really increase loss absorption capacity compared to the case of subordinated debt although that only counts as T2 capital, and increase risk taking incentives, again when compared to subordinated debt. So they certainly do not provide enough incentives for the banks to not let the conversion happen in the first place: they will not mitigate risk choices nor will

they issue additional equity to stave off conversion. In short, principal write down CoCos increase systemic risk ex ante for given levels of leverage and arguably should not qualify as (A)T1 capital.

But when the CoCos are of the convert-to-equity variety, we show that the risk shifting incentive turns negative for a sufficiently dilutive CoCo. This is because the wealth transfer itself becomes negative - while shareholders in aggregate obtain a higher residual equity upon conversion, the old shareholders must share the total residual value (i.e. old and new claims) with the new shareholders created upon conversion, which leaves the existing shareholders worse off than before conversion when the CoCo is sufficiently dilutive; this is in fact how "sufficiently dilutive" is defined. This change in residual equity, while not strictly skin in the game ex ante, is a credible threat such that the shareholders can be expected to choose risk levels that make the conversion probability smaller. As a result, the risk level chosen under sufficiently dilutive CoCos will be lower than the risk level chosen under the same amount of subordinated debt. One can even choose for CoCos to be "superdilutive" such that the risk level chosen under superdilutive CoCos will be even lower than if the same amount of additional equity was issued. This makes superdilutive CoCos more efficient than additional equity (for each unit of additional equity, one can issue a lesser amount of superdilutive CoCos) for the goal of forcing banks to comply with choosing a lower target risk level.

These results naturally lead to further questions concerning capital requirements. A corollary of our conclusions is that it is clearly not sufficient anymore to only consider asset side portfolio risk in designing capital requirements and risk weights structures: an asset side focus is not enough anymore to judge risk taking incentives for a given level of capital requirements. If CoCos are to continue to play an important role in the capital structure of banks, the level of capital requirements should also depend on how they are met. In that vein we show that some of the disadvantages of insufficiently dilutive CoCos can be offset by raising the bar higher: if inappropriate CoCo design increases risk taking incentives, that effect can be counteracted by requiring more skin in the game, i.e. by setting the requirement ratio's higher than they are set for the case of pure equity or sufficiently dilutive CoCos. We show this analytically for various CoCo designs.

These results are important in setting regulations. Basel III and the TLAC Standard were written with the focus on increasing loss absorption capacity of the financial system. To a substantial extent, this loss absorption capacity is being filled by CoCos, in particular for meeting TLAC requirements. But to achieve a more robust financial system, it is not enough to only consider loss absorption capacity. We must also consider regulation that prevents banks from choosing excessively risky actions in the first place, as the designers of Basel II fully realized when introducing risk weights. Capital regulation is meant to force banks to put more skin in the game not just to increase Loss Absorption Capacity for given risk levels, but also because equity has better (lower) risk taking incentives than debt. While CoCos are hybrids of debt and equity, it doesn't always mean that the risk levels they induce will be between those induced by debt

and equity. As we have shown, not all CoCos are created equal - some have higher risk shifting incentives than others. At the very least, the type of CoCo that is allowed to fill in AT1 capital requirements should be restricted to equity converters, and among those only CE CoCos which are sufficiently dilutive. In this way, one minimizes the chance that the loss absorption capacity has to be used in the first place.

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Annexes

1 Derivation for Eqn. (20): increasing σ and the size of the wealth transfer triggered by PWD CoCo conversion

The risk shifting incentive for a principal writedown CoCo is

$$
\begin{aligned}
RSI_{pwd}\left(\varphi\right) & = \frac{\partial\left(e_{pwd}-e_0\right)}{\partial\sigma} \\
& = \frac{\partial p^c\left(C\left[R,D+\varphi D_s|\sigma^2\right]-C\left[R,D+D_s|\sigma^2\right]\right)}{\partial\sigma} \\
& = \underbrace{\frac{\partial p^c}{\partial\sigma}\left(C\left[R,D+\varphi D_s|\sigma^2\right]-C\left[R,D+D_s|\sigma^2\right]\right)}_{CF_{pwd}}+\underbrace{p^c\frac{\partial C\left[R,D+\varphi D_s|\sigma^2\right]-C\left[R,D+D_s|\sigma^2\right]}{\partial\sigma}}_{WF_{pwd}}
\end{aligned}
$$

Where CF_{pwd} represents the conversion probability factor, and WF_{pwd} represents the wealth transfer factor. WF_{pwd} can be rewritten as the difference between the vegas of two call options that differ only in the strike price. However, we can use the mean value theorem to rewrite this difference. The derivative of vega with respect to the strike price is:

$$
V_K\left(R,K\right)=R\frac{\partial\phi\left(d_1\right)}{\partial K}=R\frac{\partial\phi\left(d_1\right)}{\partial d_1}\times\frac{\partial d_1}{\partial K}=\left[-R\phi\left(d_1\right)d_1\right]\times\left[\frac{-1}{\sigma K}\right]=\left(\frac{R\phi\left(d_1\right)}{K}\right)\left(\frac{d_1}{\sigma}\right)>0
$$

Which allows us to rewrite the wealth transfer factor in the following manner:

$$
\begin{aligned}
WF_{pwd} & = p^c\left(V\left[R,\ D+\varphi D_s|\sigma^2\right]-V\left[R,\ D+D_s|\sigma^2\right]\right) \\
& = -\ p^c\left(\left(1-\varphi\right)D_s V_K\left[R,\ D'|\sigma^2\right]\right) \\
& = -p^c\left(1-\varphi\right)D_s R\phi\left(d_1\right)\left(-\frac{\partial d_1^2}{2\partial K}\right) \\
& = p^c\left(1-\varphi\right)D_s R\phi\left(d_1\right)d_1\frac{\partial d_1}{\partial K} \\
& = -p^c\left(1-\varphi\right)\frac{D_s}{\sigma D'}R\phi\left(d_1\right)d_1 \\
& = -p^c\left(1-\varphi\right)\frac{D_s}{D'}R\phi\left(d_1\right)\frac{d_1}{\sigma} \\
& < \quad 0
\end{aligned}
$$

Note that since

$$\frac{d_1}{\sigma} = \frac{1}{\sigma^2}\left[\ln\frac{R}{D'} + r + \frac{\sigma^2}{2}\right] = \frac{1}{\sigma^2}\left[\ln\frac{R}{D'} + r\right] + \frac{1}{2} \tag{50}$$

so when σ is high, $\frac{d_1}{\sigma}$ goes to 0 but d_1 will grow with σ and thus $\phi(d_1)$ will fall. Hence $\phi(d_1)\frac{d_1}{\sigma}$ will fall as σ rises. Thus, for sufficiently high σ levels, given everything else, we have that WF_{pwd} goes to zero while CF_{pwd} increases in σ. Similarly, WF_{pwd} becomes smaller as the leverage D'/R becomes higher and as the conversion probability p^c becomes lower.

2 Derivation of Eqn. (21): the effect of an increase in φ on the risk-shifting incentive from PWD CoCos

$$\frac{\partial}{\partial\varphi}\left[\frac{\partial p^c}{\partial\sigma}W_{pwd} + p^c\frac{\partial W_{pwd}}{\partial\sigma}\right]$$

$$= \frac{\partial p^c}{\partial\sigma}\frac{\partial\left(C\left[R, D + \varphi D_s|\sigma^2\right] - C\left[R, D + D_s|\sigma^2\right]\right)}{\partial\varphi} + p^c\frac{\partial\left(V\left[R,\ D + \varphi D_s|\sigma^2\right] - V\left[R,\ D + D_s|\sigma^2\right]\right)}{\partial\varphi}$$

$$= \underbrace{\frac{\partial p^c}{\partial\sigma}\frac{\partial C\left[R, D + \varphi D_s\right]}{\partial\varphi}}_{\partial CF_{pwd}/\partial\varphi} + \underbrace{p^c\frac{\partial V\left[R,\ D + \varphi D_s\right]}{\partial\varphi}}_{\partial WF_{pwd}/\partial\varphi}$$

$$= \underbrace{-\frac{\partial p^c}{\partial\sigma}\exp(-r)\,\Phi\left(d_2\left(D + \varphi D_s\right)\right)D_s}_{\partial CF_{pwd}/\partial\varphi} + \underbrace{p^c V_K D_s}_{\partial WF_{pwd}/\partial\varphi}$$

$$= \underbrace{-\frac{\partial p^c}{\partial\sigma}\exp(-r)\,\Phi\left(d_2\left(D + \varphi D_s\right)\right)D_s}_{\partial CF_{pwd}/\partial\varphi} + \underbrace{p^c\frac{R\phi\left(d_1\left(D + \varphi D_s\right)\right)D_s}{D + \varphi D_s}\frac{d_1}{\sigma}}_{\partial WF_{pwd}/\partial\varphi}$$

$$= \underbrace{-\frac{\partial p^c}{\partial\sigma}\exp(-r)\,\Phi\left(d_2\left(D + \varphi D_s\right)\right)D_s}_{\partial CF_{pwd}/\partial\varphi} + \underbrace{p^c\phi\left(d_1\left(D + \varphi D_s\right)\right)D_s\frac{R}{D + \varphi D_s}\frac{d_1}{\sigma}}_{\partial WF_{pwd}/\partial\varphi}$$

where the notations $d_1\left(D + \varphi D_s\right)$ and $d_2\left(D + \varphi D_s\right)$ refer to d_1 and d_2 with strike price $D + \varphi D_s$.

3 Rewriting of Eqn. (23)

$$
\begin{aligned}
RSI_{ce} &= \frac{\partial}{\partial\sigma}\left\{\frac{p^c}{1+\psi D_s}C\left[R,\,D\right]\right\} - \frac{\partial}{\partial\sigma}\left\{p^c C\left[R,\,D+D_s\right]\right\} \\
&= \frac{1}{1+\psi D_s}\left[p^c V\left[R,\,D\right] + \frac{\partial p^c}{\partial\sigma}C\left[R,\,D\right]\right] - \left[p^c V\left[R,\,D+D_s\right] + \frac{\partial p^c}{\partial\sigma}C\left[R,\,D+D_s\right]\right] \\
&= p^c\left[\frac{V\left[R,\,D\right]}{1+\psi D_s} - V\left[R,\,D+D_s\right]\right] + \frac{\partial p^c}{\partial\sigma}\left[\frac{C\left[R,\,D\right]}{1+\psi D_s} - C\left[R,\,D+D_s\right]\right] \\
&= \underbrace{\frac{\partial p^c}{\partial\sigma}\left[\frac{C\left[R,\,D\right]}{1+\psi D_s} - C\left[R,\,D+D_s\right]\right]}_{CF_{ce}} + \underbrace{p^c\left[\frac{V\left[R,\,D\right]}{1+\psi D_s} - V\left[R,\,D+D_s\right]\right]}_{WF_{ce}}
\end{aligned}
$$

4 Proof for Eqn. (25)

The equation for RSI_{ce} is

$$
RSI_{ce} = \underbrace{\frac{\partial p^c}{\partial\sigma}\left[\frac{C\left[R,\,D\right]}{1+\psi D_s} - C\left[R,\,D+D_s\right]\right]}_{CF_{ce}} + \underbrace{p^c\left[\frac{V\left[R,\,D\right]}{1+\psi D_s} - V\left[R,\,D+D_s\right]\right]}_{WF_{ce}}
$$

This means that the threshold level of ψ that makes the risk shifting incentive exactly zero lies somewhere within the interval $\left[0,\,\tilde{\psi}\right]$ because at $\tilde{\psi}$, $RSI_{ce} < 0$ Since CF_{ce} and WF_{ce} are generally of opposite signs, we need only choose a ψ that makes CF_{ce} positive and exactly offsets the negative value of WF_{ce}. In other words, choose ψ such that $p^c\left[\frac{V[R,\,D]}{1+\psi D_s} - V\left[R,\,D+D_s\right]\right] = \frac{\partial p^c}{\partial\sigma}\left[\frac{C[R,\,D]}{1+\psi D_s} - C\left[R,\,D+D_s\right]\right]$. Let us call this value $\psi_{RSI=0}$.

We claim that $\psi_{RSI=0} < \tilde{\psi}$. The expression for $\tilde{\psi}$ is

$$
\tilde{\psi} = \frac{1}{D_s}\left\{\frac{C\left[R,\,D|\sigma^2\right]}{C\left[R,\,D+D_s|\sigma^2\right]} - 1\right\} \tag{51}
$$

On the other hand, the expression for $\psi_{RSI=0}$ is

$$
\psi_{RSI=0} = \frac{1}{D_s}\left\{\frac{\frac{\partial p^c}{\partial\sigma}C\left[R,\,D\right] + p^c V\left[R,\,D\right]}{\frac{\partial p^c}{\partial\sigma}C\left[R,\,D+D_s\right] + p^c V\left[R,\,D+D_s\right]} - 1\right\} \tag{52}
$$

which can be rewritten as

$$
\psi_{RSI=0} = \frac{1}{D_S}\left\{\frac{C\left[R,\,D\right]\left(\frac{\partial p^c}{\partial\sigma} + p_c\frac{V[R,\,D]}{C[R,\,D]}\right)}{C\left[R,\,D+D_s\right]\left(\frac{\partial p^c}{\partial\sigma} + p_c\frac{V[R,\,D+D_s]}{C[R,\,D+D_s]}\right)} - 1\right\} \tag{53}
$$

$\tilde{\psi} = \psi_{RSI=0}$ if and only if $\frac{V[R,\ D]}{C[R,\ D]} = \frac{V[R,\ D+D_s]}{C[R,\ D+D_s]}$. However, we can write $\frac{V[R,\ D+D_s]}{C[R,\ D+D_s]}$ as follows:

$$\frac{V[R,\ D+D_s]}{C[R,\ D+D_s]} = \frac{V[R,\ D] + V_K D_s}{C[R,\ D] + C_K D_s} > \frac{V[R,\ D]}{C[R,\ D]} \tag{54}$$

where V_K is the derivative of vega with respect to the strike price, and C_K is the derivative of the call option value with respect to the strike price. The inequality follows from $C_K < 0 < V_K$: the value of a call option falls when the strike price rises, while the vega of a call option rises when the strike price rises. Therefore we have shown that $\psi_{RSI=0} < \tilde{\psi}$, as claimed.

5 Derivation for Eqn. (28): concavity of vega for sufficiently high σ

From Eqn. (19), the value of residual equity is always increasing in σ. However, vega itself does not exhibit that sort of behavior. A closer look at vega shows that the sign of its derivative with respect to σ changes sign at $\sigma^2 = 2\left(\ln\frac{R}{K} + r\right)$:

$$\begin{aligned}
\frac{\partial V[R,\ K]}{\partial \sigma} &= R\phi'(d_1)\frac{\partial d_1}{\partial \sigma} \\
&= -R\phi(d_1)d_1\left(1 - \frac{d_1}{\sigma}\right) \\
&< 0 \ for \ \sigma^2 > 2\left(\ln\frac{R}{K} + r\right)
\end{aligned} \tag{55}$$

Where $d_1 = \frac{1}{\sigma}\left(\ln\frac{R}{K} + r + \frac{1}{2}\sigma^2\right)$ and K is the strike price. Therefore, when σ^2 exceeds $2\left(\ln\frac{R}{K} + r\right)$, vega is concave in σ.

A closer look at the inflection point $\sigma^2 = 2\left(\ln\frac{R}{K} + r\right)$ shows that it is increasing in R. This means that during times of low R (e.g. crisis periods), it takes a lower σ to reach the high-risk area. This is true regardless of the role that D_s plays in the capital structure of a bank.